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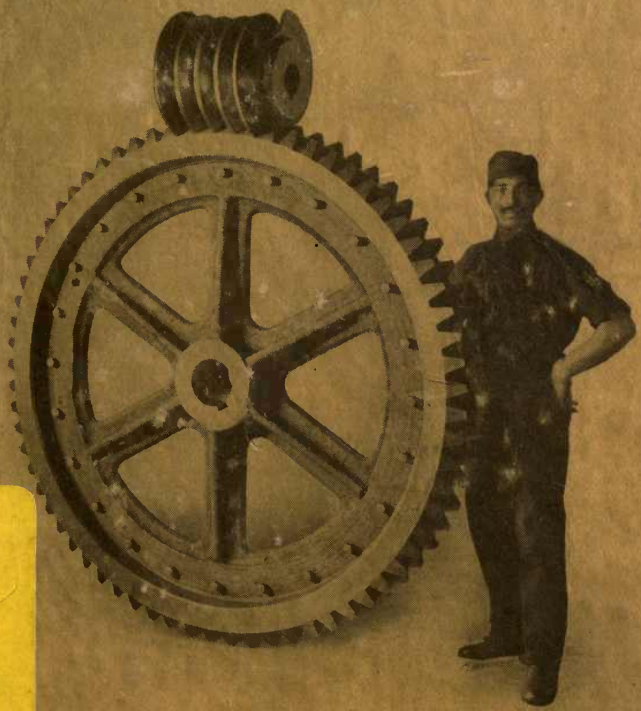
WORM GEARING

CALCULATION OF WORM GEARS – HOBS SELF-LOCKING WORM GEARING

FOURTH REVISED AND ENLARGED EDITION



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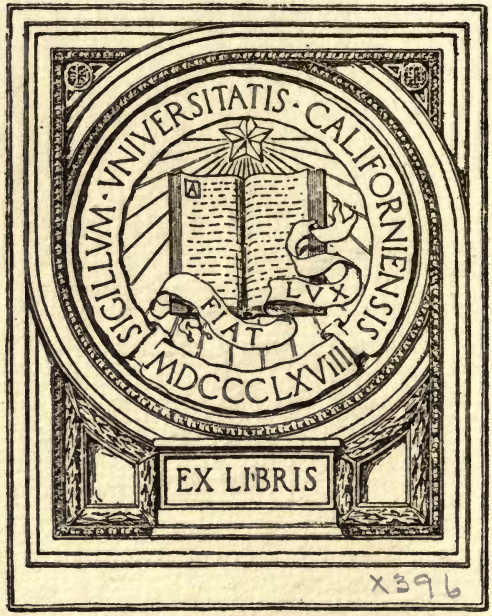
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NUMBER 1

WORM GEARING

FOURTH REVISED AND ENLARGED EDITION

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CHAPTER I

CALCULATING THE DIMENSIONS OF WORM GEARING*

The present chapter contains a compilation of rules for the calculation of the dimensions of worm gearing, expressed with as much simplicity and clearness as possible. No attempt has been made to give rules for estimating the strength or durability of worm gearing, although the question of durability, especially, is the determining factor in the design of worm gearing. If the worm and wheel are so proportioned as to have a reasonably long life under normal working conditions, it may be taken for granted that the teeth are strong enough for the load they have to bear. No simple rules have ever been proposed for proportioning worm gearing to suit the service it is

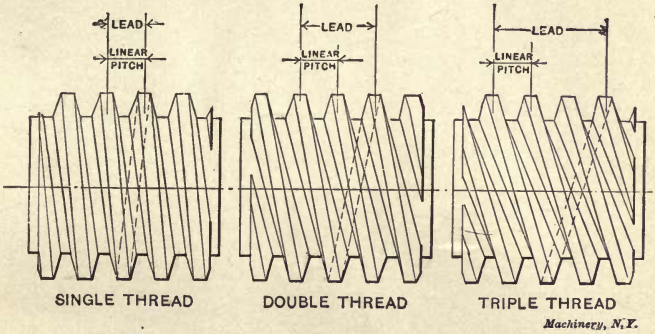

Fig. 1. Distinction between the Terms Lead and Linear Pitch as Applied to Worms

designed for. Judgment and experience are about the only factors the designer has for guidance. In Europe, a number of builders are regularly manufacturing worm drives, guaranteed for a given horse-power at a given speed. The dimensions of these drives are not made public, however; they would doubtless be of great value for purposes of comparison if they could be obtained. In the absence of these or other practical data, this phase of the subject has, of necessity, not been entered upon.

Definitions and Rules for Dimensions of the Worm

In giving names to the dimensions of the worm, there is one point in which there is sometimes confusion. This relates to the distinction between the terms "pitch" and "lead." In the following we will adhere to the nomenclature indicated in Fig. 1. Here are shown three worms, the first single-threaded, the second double-threaded, and the

* MACHINERY, August, 1907.

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last triple-threaded. As shown, the word "lead" is assumed to mean the distance which a given thread advances in one revolution of the worm, while by "pitch," or more strictly, "linear pitch," we mean the distance between the centers of two adjacent threads. As may be clearly seen, the lead and linear pitch are equal for a single-threaded worm. For a double-threaded worm the lead is twice the linear pitch, and for a triple-threaded worm it is three times the linear pitch. From this we have:

RULE 1. *To find the lead of a worm, multiply the linear pitch by the number of threads.*

It is understood, of course, that by the number of threads is meant, not the number of threads per inch, but the number of threads in the whole worm—one, if it is single-threaded, four, if it is quadruple-threaded, etc. Rule 1 may be transposed to read as follows:

RULE 2. *To find the linear pitch of a worm, divide the lead by the number of threads.*

The standard form of worm thread, measured in an axial section

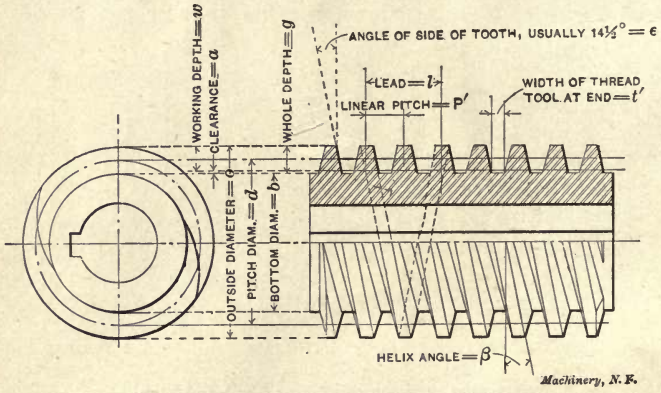


Fig. 2. Nomenclature of Worm Dimensions

as shown in Fig. 2, has the same dimensions as the standard form of involute rack tooth of the same linear or circular pitch. It is not of exactly the same shape, however, not being rounded at the top, nor provided with fillets. The thread is cut with a straight-sided tool, having a square, flat end. The sides have an inclination with each other of 29 degrees, or 14½ degrees with the center line. The following rules give the dimensions of the teeth in an axial section for various linear pitches. For nomenclature, see Fig. 2.

RULE 3. *To find the whole depth of the worm tooth, multiply the linear pitch by* 0.6866.

RULE 4. *To find the width of the thread tool at the end, multiply the linear pitch by* 0.31.

RULE 5. *To find the addendum or height of worm tooth above the pitch line, multiply the linear pitch by* 0.3183.

RULE 6. *To find the outside diameter of the worm, add together the pitch diameter and twice the addendum.*

RULE 7. *To find the pitch diameter of the worm, subtract twice the addendum from the outside diameter.*

RULE 8. *To find the bottom diameter of the worm, subtract twice the whole depth of tooth from the outside diameter.*

RULE 9. *To find the helix angle of the worm and the gashing angle of the worm-wheel tooth, multiply the pitch diameter of the worm by* 3.1416, *and divide the product by the lead; the quotient is the cotangent of the tooth angle of the worm.*

Rules for Dimensioning the Worm-Wheel

The dimensions of the worm-wheel, named in the diagram shown in Fig. 3, are derived from the number of teeth determined upon for it,

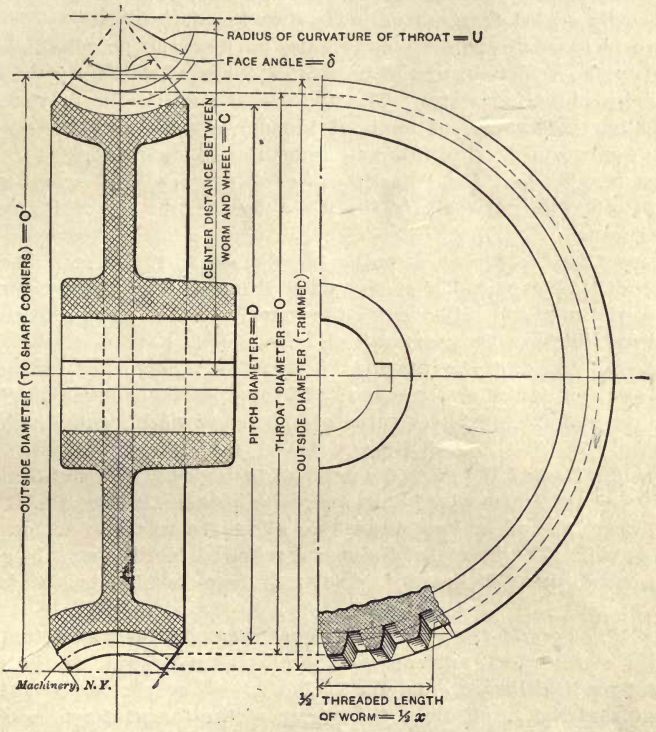


Fig. 3. Nomenclature of Worm-wheel Dimensions

and the dimensions of the worm with which it is to mesh. The following rules may be used:

RULE 10. *To find the pitch diameter of the worm-wheel, multiply the number of teeth in the wheel by the linear pitch of the worm, and divide the product by* 3.1416.

RULE 11. *To find the throat diameter of the worm-wheel, add twice the addendum of the worm tooth to the pitch diameter of the worm-wheel.*

RULE 12. *To find the radius of curvature of the worm-wheel throat, subtract twice the addendum of the worm tooth from half the outside diameter of the worm.*

The face angle of the wheel is arbitrarily selected; 60 degrees is a good angle, but it may be made as high as 80 or even 90 degrees, though there is little advantage in carrying the gear around so great a portion of the circumference of the worm, especially in steep pitches.

RULE 13. *To find the diameter of the worm-wheel to sharp corners, multiply the throat radius by the cosine of half the face angle, subtract this quantity from the throat radius, multiply the remainder by 2, and add the product to the throat diameter of the worm-wheel.*

If the sharp corners are flattened a trifle at the tops, as shown in Figs. 3 and 5, this dimension need not be figured, "trimmed diameter" being easily scaled from an accurate drawing of the gear.

There is a simple rule which, rightly understood, may be used for obtaining the velocity ratio of a pair of gears of any form, whether spur, spiral, bevel, or worm. The number of teeth of the driven gear, divided by the number of teeth of the driver, will give the velocity ratio. For worm gearing this rule takes the following form:

RULE 14. *To find the velocity ratio of a worm and worm-wheel, divide the number of teeth in the wheel by the number of threads in the worm.*

Be sure that the proper meaning is attached to the phrase "number of threads" as explained before under Rule 1. The revolutions per minute of the worm, divided by the velocity ratio, gives the revolutions per minute of the worm-wheel.

RULE 15. *To find the distance between the center of the worm-wheel and the center of the worm, add together the pitch diameter of the worm and the pitch diameter of the worm-wheel, and divide the sum by 2.*

RULE 16. *To find the pitch diameter of the worms, subtract the pitch diameter of the worm-wheel from twice the center distance.*

The worm should be long enough to allow the wheel to act on it as far as it will. The length of the worm required for this may be scaled from a carefully-made drawing, or it may be calculated by the following rule:

RULE 17. *To find the minimum length of worm for complete action with the worm-wheel, subtract four times the addendum of the worm thread from the throat diameter of the wheel, square the remainder, and subtract the result from the square of the throat diameter of the wheel. The square root of the result is the minimum length of worm advisable.*

The length of the worm should ordinarily be longer than the dimension thus found. Hobs, particularly, should be long enough for the largest wheels they are ever likely to be called upon to cut.

Departures from the Above Rules

The throat diameter of the wheel and the center distance may have to be altered in some cases from the figures given by the preceding

rules. If worm-wheels with small numbers of teeth are made to the dimensions given, it will be found that the flanks of the teeth will be partly cut away by the tops of the hob teeth, so that the full bearing area is not available. The matter becomes serious when there are less than 25 teeth in the worm-wheel. There are two ways of avoiding the difficulty. One of them is to increase the included angle of the sides of the thread tool. This departure from standard form, however, may be avoided by an increase in the throat diameter of the wheel, and consequently in the center distance. Discussions of this subject will be found in "Formulas in Gearing," and "Practical Treatise on Gearing," both published by the Brown & Sharpe Mfg. Co., Providence, R. I.

On the other hand, some designers claim to get better results in efficiency and durability by making the throat diameter of the worm-wheel *smaller* than standard, where it is possible to do so without too much under-cutting. A discussion of this subject will be found in Chapter IV of this treatise. In no case, however, should the throat diameter ever be made so small as to produce more interference than is met with in a standard 25-tooth worm-wheel.

Two Applications of Worm Gearing

Worm-wheels are used for two purposes. They may be employed to transmit power where it is desired to make use of the smoothness of action which they give, and the great reduction in velocity of which they are capable; instances of this application of worm gearing are found in the spindle drives of gear cutters and other machine tools. They are also used where a great increase in the effective power is required; in this case advantage is generally taken of the possibility of making the gearing self-locking. Such service is usually intermittent or occasional, and the matter of waste of power is not of so great importance as in the first case. Examples of this application are to be found in the adjustments of a great many machine tools, in training and elevating gearing for ordnance, etc. Calculations for the general design of this class of gearing will be treated separately. (See Chapter VI.) In the case of elevator gearing and worm feeds for machinery, the functions of the gearing are, in a measure, a combination of those in the two applications.

Examples of Worm Gearing Figured from the Rules

To show how the rules given above may be applied, we will work out two examples. The first of these is for a light machine tool spindle drive, in which power is to be transmitted continuously. It is determined that the velocity ratio shall be 8 to 1, and that the proper linear pitch to give the strength and durability required shall be about ¾ inch; the center distance is required to be 5 inches exactly. This case comes under the first of the two applications just described.

Assume, for instance, 32 teeth in the wheel, and a quadruple-thread worm. We will figure the gearing with these assumptions, and see if it appears to have practical dimensions.

The pitch diameter of the worm-wheel by Rule 10 is found to be

$$\frac{32 \times \frac{3}{4}}{3.1416} = 7.6394 \text{ inches.}$$

The pitch diameter of the worm by Rule 16 is found to be

$$(2 \times 5) - 7.6394 = 2.3606 \text{ inches.}$$

The addendum of the worm thread by Rule 5 is found to be

$$0.3183 \times \tfrac{3}{4} = 0.2387 \text{ inch.}$$

The outside diameter of the worm by Rule 6 is found to be

$$2.3606 + (2 \times 0.2387) = 2.8380 \text{ inches.}$$

For transmission gearing the angle of inclination of the worm thread should be not less than 18 degrees or thereabouts, and the nearer 30 or even 40 degrees it is, the more efficient will it be. From Rule 1 we find the lead to be $4 \times \frac{3}{4} = 3$ inches.

The helix angle of the worm thread is found from Rule 9, $2.3606 \times 3.1416 \div 3 = 2.4722 = \text{cot. } 22$ degrees, approximately. This angle will give fairly satisfactory results. The calculations are not carried any further with this problem, whose other dimensions are determined from those just found. In the following case, however, all the calculations are made.

For a second problem let it be required to design worm feed gearing for a machine to utilize a hob already in stock. This hob is double-threaded, ½ inch linear pitch, and 2½ inches diameter. The center distance of the gearing is immaterial, but it is decided that the worm-wheel ought to have about 45 teeth to bring the ratio right. The only calculations made are those necessary for the dimensions which would appear on the shop drawing.

To find the lead, use Rule 1: $0.5 \times 2 = 1.0$ inch.

To find the whole depth of the worm tooth, use Rule 3: $0.5 \times 0.6866 = 0.3433$ inch.

To find the addendum, use Rule 5: $0.5 \times 0.3183 = 0.15915$ inch.

To find the pitch diameter of the worm, use Rule 7: $2.5 - 2 \times 0.15915 = 2.1817$ inches.

To find the bottom diameter of the worm, use Rule 8: $2.5 - 2 \times 0.3433 = 1.8134$ inch.

To find the gashing angle of the worm-wheel, use Rule 9: $2.18 \times 3.14 \div 1 = 6.845 = \text{cot. } 8$ degrees 20 minutes, about.

To find the pitch diameter of the worm-wheel, use Rule 10: $45 \times 0.5 \div 3.1416 = 7.1620$ inches.

To find the throat diameter of the worm-wheel, use Rule 11: $7.1620 + 2 \times 0.15915 = 7.4803$ inches.

To find the radius of the throat of the worm-wheel, use Rule 12: $(2.5 \div 2) - (2 \times 0.15915) = 0.9317$ inch.

The angle of face may be arbitrarily set at, say, 75 degrees, in this case. The "trimmed diameter" is scaled from an accurate drawing, and proves to be 7.75 inches.

To find the distance between centers of the worm and wheel, use Rule 15: $(2.1817 + 7.1620) \div 2 = 4.6718$ inches.

To find the minimum length of threaded portion of the worm, use

Rule 17: $7.4803 - 4 \times 0.15915 = 6.8437$

$\sqrt{7.4803^2 - 6.8437^2} = 3$ inches, approximately.

It will be noted that the ends of the threads in Fig. 2 are trimmed at an angle instead of being cut square down, as in Fig. 1. This gives a more finished look to the worm. It is easily done by applying the sides of the thread tool to the blank just before threading, or it may be done as a separate operation in preparing the blank, which will in either case have the appearance shown in Fig. 4. The small diameters at either end of the blank in Fig. 4 should, in any event, be turned exactly to the bottom diameter shown in Fig. 2, and obtained by Rule 8. This is of great assistance to the man who threads the worm,

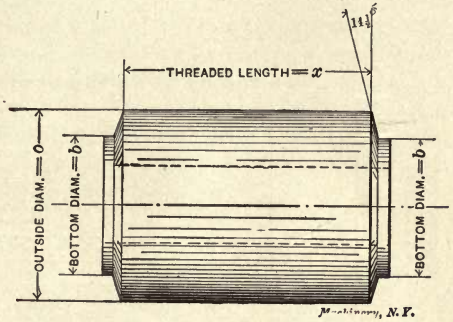


Fig. 4. Shape of Blank for Worm

as he knows that the threads are sized properly as soon as he has cut down to this diameter with the end of his thread tool. This always supposes, of course, that the thread tool is accurately made.

Formulas for the Design of Worm Gearing

For the convenience of those who prefer to have their rules compressed into formulas, they are so arranged in the following. The reference letters used are as follows:

N = number of teeth in worm-wheel.
n = number of teeth or threads in worm.
P' = circular pitch of wheel and linear pitch of worm.
l = lead of worm.
g = whole depth of worm tooth.
t' = width of the thread tool at the end.
s = addendum or height of worm tooth above pitch line.
o = outside diameter of the worm.
d = pitch diameter of the worm.
b = bottom or root diameter of the worm.
β = helix angle of worm and gashing angle of wheel.
δ = face-angle of worm-wheel.
D = pitch diameter of the worm-wheel.
O = throat diameter of the worm-wheel.
O' = diameter of the worm-wheel to sharp corners.
U = radius of curvature of the worm-wheel throat.

R = velocity ratio.
C = distance between centers.
x = threaded length of worm.

$$l = n \times P' \quad (1)$$
$$P' = l \div n \quad (2)$$
$$g = 0.6866\,P' \quad (3)$$
$$t' = 0.31\,P' \quad (4)$$
$$s = 0.3183\,P' \quad (5)$$
$$o = d + 2s \quad (6)$$
$$d = o - 2s \quad (7)$$
$$b = o - 2g \quad (8)$$

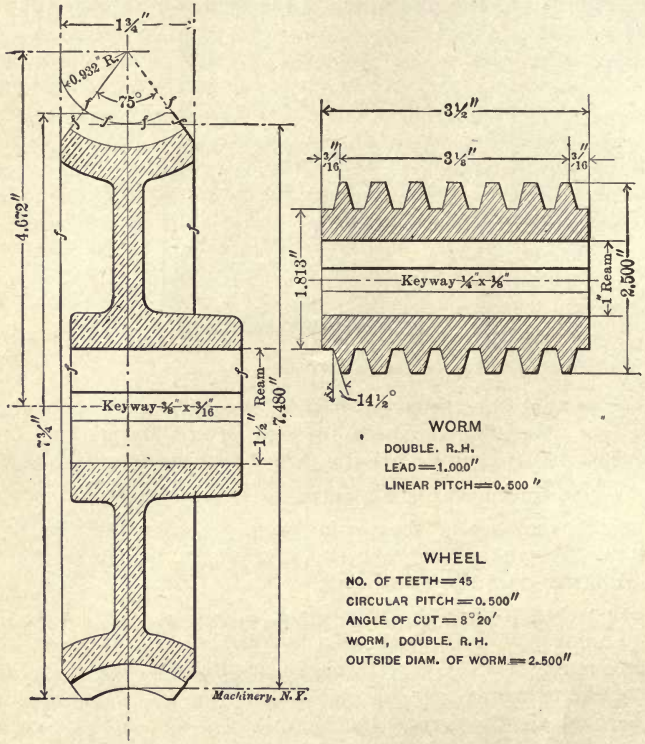


Fig. 5. Model Drawing of Worm and Worm-wheel

$$\text{Cotangent } \beta = 3.1416d \div l \quad (9)$$
$$D = N\,P' \div 3.1416 \quad (10)$$
$$O = D + 2s \quad (11)$$
$$U = \tfrac{1}{2}o - 2s \quad (12)$$
$$O' = 2\,(U - U\cos\delta/2) + O \quad (13)$$
$$R = N \div n \quad (14)$$
$$C = (D + d) \div 2 \quad (15)$$
$$d = 2C - D \quad (16)$$
$$\text{Minimum value of } x = \sqrt{O^2 - (O - 4s)^2} \quad (17)$$
Width of worm-wheel at root of teeth = $0.6o$ (approx.) (18)

A model drawing of a worm-wheel and worm, properly dimensioned, is shown in Fig. 5. This drawing follows, in general, the model drawings shown by Mr. Burlingame in the August, 1906, issue of MACHINERY, taken from the drafting-room practice of the Brown & Sharpe Mfg. Co. In cases where the worm-wheel is to be gashed on the milling machine before hobbing, the angle at which the cutter is set should also be given. This is the same as the angle of worm tooth found by Rule 9. In cases where the wheel is to be hobbed directly from the solid by a positively geared hobbing machine, this information is not needed. It might be added that it is impracticable with worm-wheels having less than 16 or 18 teeth to gash the wheel, and then hob it when running freely on centers, if the throat diameter has been determined by Rule 11.

When worms have a large helix angle (15 degrees or more), the dimensions of the tooth should be measured at right angles to the helix. In such cases, the following changes should be made in the formulas just given.

Let P'_n = normal circular pitch = $P' \cos \beta$.

Formulas (3), (4), and (5), and the corresponding rules, should then be written as follows:

$$g = 0.6866\, P'_n \qquad (3)$$

$$t' = 0.31\, P'_n \qquad (4)$$

$$s = 0.3183\, P'_n \qquad (5)$$

When these changes are made, all the other formulas will give correct results when used in their original form.

CHAPTER II

HOBS FOR WORM-GEARS*

If a worm and gear of standard proportions are brought into mesh, we have at the bottom of both the thread of the worm and teeth of the gear a clearance equal to one-tenth of the thickness of the thread or tooth at the pitch line. The clearance at the root of the gear tooth is obtained by enlarging the hob over the diameter of the worm, by an amount equal to two clearances, while the clearance of the tooth in the thread bottom is taken care of by the proper sizing of the gear blank.

While it may be customary practice to make the hob an exact duplicate of the worm except in the one item of outside diameter, a hob proportioned as suggested in Fig. 7 is recommended as one that will give much more satisfactory results, and be found to be well worth any additional trouble in construction required beyond that for the style ordinarily used. The peculiar feature of this hob is that it is an

* MACHINERY, September, 1907.

exact opposite of the worm with respect to the proportions of the thread shape; the depth below the pitch line in one case being equal to the height above the pitch line in the other. The object of this is to have a hob that will form the complete outline of the tooth and make it absolutely certain that the standard proportions of tooth and clearance are obtained. Thus, should the diameter of the blank be large, the hob will trim off the top of the gear teeth to the proper length, when the proper center distance is maintained.

There is another point that is generally overlooked, and that is the necessity for having the corners of the thread rounded over, and for providing a liberal fillet at the root of the thread. The radii of the rounded corner and the fillet may be as large as the clearance will allow, which would be one-twentieth of the circular pitch of the thread.

The effect that this fillet and rounded thread have on the shape of

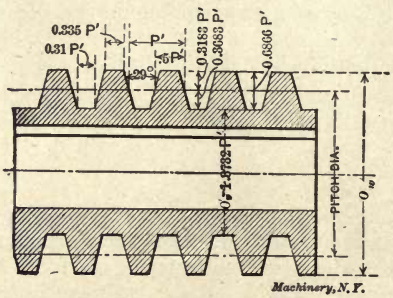


Fig. 6. Dimensions of Worm

the tooth is something that greatly increases the quality of the gear and the strength of each individual tooth. The rounded corner on the thread points does away with any tendency to scratch the surface of the tooth in the cutting action, and leaves a much larger fillet at the root, greatly increasing the strength. The fillet at the bottom of the thread rounds off the top of the tooth in the worm-gear, removing any burrs, and leaving a nicely finished product. This fillet also removes the dangerous tendency of the hob to develop cracks in the hardening process—a common source of trouble even where care is taken. Fig. 6 shows the proportions of the worm in comparison with the hob in Fig. 7.

In forming the hob, much can be gained by making a special form tool of correct proportion that will leave no chance for error; the only dimension needing care then, is the diameter. Such a tool is shown in Fig. 9. The figure is dimensioned by formulas, so that a tool for any pitch can be easily proportioned from it. This tool may be made by using a gear caliper without resorting to the protractor, or the protractor may be used in laying out the angle. This tool may be made without side clearance, providing that the sides incline in the same direction and at the same angle that the thread takes, but under ordinary circumstances, where only one hob is to be made, little is gained by having no side clearance. Clearance may be made

from 5 to 10 degrees from the angle of the thread. Grinding a tool like this of course changes its form, so it must not be used indefinitely in making large numbers of similar hobs.

Number of Flutes in Hobs

The number of flutes that should be provided in the hob is a point on which very little is said, various authorities differing widely. Where the hob is to be used in an automatic hobbing machine in which the hob and blank are positively geared together, the number of flutes may be a comparatively small number as compared with a hob that is to be used in connection with ordinary processes of hobbing worm gears. In the process in which the previously gashed worm-gear blank is swung loosely on centers and revolved by the hob as the latter rotates, the hob should have a larger number of flutes.

A rule that checks up well with present practice is as follows:

To find the number of flutes in a hob, multiply the diameter of the hob by three, and divide by twice the circular pitch.

The above rule gives suitable results on hobs for general purposes.

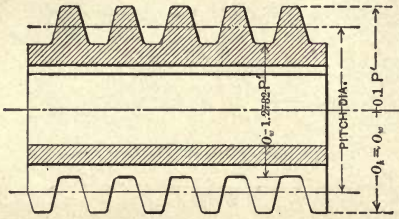


Fig. 7. Dimensions of Hob

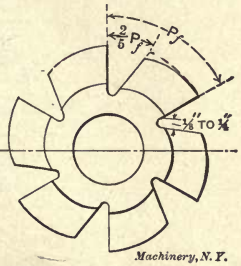


Fig. 8. Data for Fluting Hob

When the result gives an odd number of teeth, take the next smaller even number, to facilitate calipering.

Some authorities on worm-gearing state that the number of flutes in a hob should in no case be an exact multiple of the number of threads. Their reason for this rule is that the hob so gashed will produce a much smoother tooth and one nearer correct in shape, because no tooth in the hob passes the same tooth in the gear twice in succession, so that any little imperfections in shape of the individual hob teeth are counteracted by one another. Another authority is strong in his advice not to have the circumferential distance from flute to flute equal to or equally divisible by the circular pitch, for the same reason as stated regarding the former rule. From these statements, it is seen that to obtain a rule that would be at once simple and yet take all conditions into consideration, would be a difficult proposition. It seems, however, that only the first of these two rules is a logical one. Owing to the fact that hobs have teeth only, instead of full surfaces matching the worm, the curved outlines of the wheel teeth are merely approximated by a series of tangents. If the number of flutes in the hob is a multiple of the number of threads, the hob teeth will "track" after each other, giving wheel teeth

only roughly approximated by a comparatively small number of long tangents.

The cutter used in gashing the hob should be about ⅛ inch thick at the periphery for hobs of ordinary pitch, while for those of coarser pitch a cutter ¼ inch thick would be much better. The width of the gash at the periphery of the hob should be about two-fifths the pitch of the flutes. The cutter should be sunk into the blank so that it reaches from 3/16 to ¼ inch below the root of the thread. Fig. 8 shows an end view of a hob gashed according to these rules.

Where a hob is to be used to any great extent, and is subject to much wear, it would be advisable to increase the diameter above the

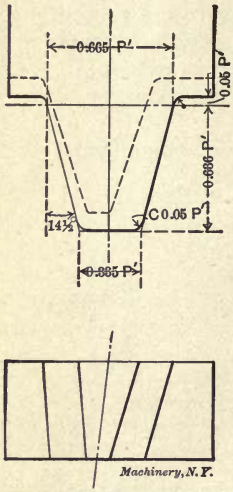


Fig. 9. Dimensions of Tool for Threading Hob

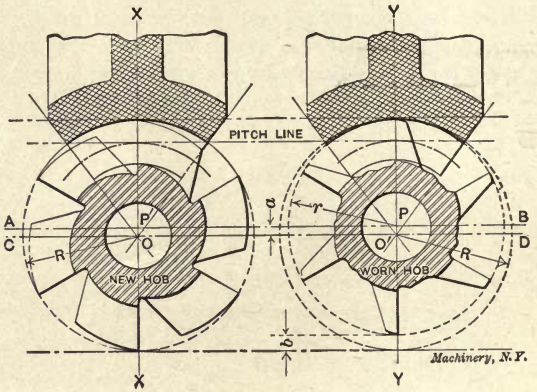


Fig. 10. The Difference in Shape of Teeth Cut by New and Old Hobs

dimensions given from 0.010 to 0.030 inch according to its diameter and pitch, to allow for decrease in diameter due to the relief, and caused by grinding back the cutting face in sharpening.

Hobs are generally fluted parallel with the axis, but it is obvious that they should be gashed on a spiral at right angles with the thread helix in order that the cutting face may be presented with theoretical correctness; but the trouble encountered in relieving the teeth on the ordinary backing off attachment is the cause of the common mode of fluting. When the pitch or lead is coarse in comparison with the pitch diameter of the hob, so that the angle is correspondingly steep, it may be best to flute on the normal helix, and if the hob cannot be machine relieved, it may be backed off by hand.

The amount of relief depends much on the use for which the hob is intended. A hand hob for hobbing a gear in position may be made with little or no relief, while hobs used on hobbing machines may have much more relief than those used on the milling machine.

CHAPTER III

SUGGESTED REFINEMENT IN THE HOBBING OF WORM-WHEELS*

At the left of Fig. 10 is a sectional view showing a hob in the act of putting the last finishing touches on a worm-wheel. The hob is supposed to be a new one and is shown in the condition it is in when first received from the makers. At the right of Fig. 10 is shown the same hob putting the finishing touches on a worm-wheel similar to that in the first case. The hob in this case is represented as having been in use for a considerable time, and having been ground down to the last extremity, ready to be discarded for a new one. A study of this cut will show that if the hob is made in the first place to properly match the worm which is to drive the wheel, it will not, when worn, cut exactly the proper form of tooth in the blank to mesh with that worm. The teeth are cut to the same depth in each case, this being necessary in order to make a proper fit with the worm, which is the same in each case and is set at the same center distance. The grinding away of the worn hob has reduced its diameter by an amount indicated by dimension *b*. Its center is therefore at *P* on the line *A B*, which is offset by a distance represented by dimension *a* from the line *C D* on which the center *O* of the new hob is located. This reduction in diameter as the hob is ground away from time to time, so evidently follows from the construction of the relieved hob, that it scarcely needs to be explained.

It is said of relieved hobs that they can be ground without changing their shape. This is true so far as the outline of the cutting edge is concerned, but it will be evident on examining the conditions shown at the right hand of Fig. 10, that whatever the outline of the cutting edges, a new hob of radius *R* will not cut exactly the same shape teeth in the blank as the worn hob with radius *r*. The elements of the tooth surface it generates are struck from a center *P*, removed by dimension *a* from center *O'* which is the location of the axis of the worm with which it meshes.

It is possible, and perhaps practicable, to overcome this slight error; that is, to so design and use the hob that it will cut as correct teeth when worn as when new. In Fig. 11, dotted line *A A* represents the outlines of a new hob in the act of finishing the worm-wheel shown. Were a hob, ground as shown at the right of Fig. 10, to be substituted on the arbor for this new hob, without altering the adjustment of the machine except to move the hob endwise and bring it in contact with the teeth of the wheel on one side, this hob would be represented in Fig. 11 by the full line *B B*. It is evident that the left-hand cutting edges of this hob coincide (to the depth they extend into the wheel) with those of the new hob represented by outline *A A*. They will,

* MACHINERY, May, 1907.

therefore, so far as they extend, cut identically similar and correct tooth curves with the new hob.

Teeth cut with this worn hob would, however, evidently have two faults. The space would be too narrow at the pitch line by a distance measured by dimension *m*, and they would not be cut deep enough in the blank by a distance measured by dimension *n*. Our problem is to so alter the design and application of the hob, that, even when worn, we can cut the teeth deep enough and the space wide enough.

Fig. 12 shows these conditions fulfilled. Dotted line *C C* shows the outline of the proposed hob when new. The only difference between the proposed hob and the regular one, whose outlines are shown by the dotted line *A A* in Fig. 11, is that the teeth have been lengthened by an amount equal to dimension *o*. The hob is fed in as was the case with the new hob in Fig. 11 until the distance between its center line and that of the blank is the same as that between the center line of the worm and the wheel in the finished machine. The increase in

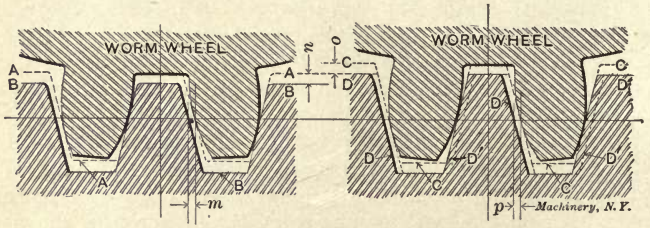


Fig 11. Cutting Action of Ordinary Hob at Fixed Center Distance, when New and when Worn

Fig. 12. Cutting Action of Proposed Hob, when New and when Old

radius, then, by an amount *o*, makes the hob cut a clearance deeper than is necessary by that amount. In a spur gear this would doubtless be a bad thing, since it would make the tooth slenderer and therefore weaker. A worm-gear, however, if designed to be sufficiently durable for continuous use, is almost certain to be several times stronger than necessary, so that the slight weakening involved in the change is not of great importance. When the hob is worn to the shape shown by the full outline *D D*, the hob is evidently of the same diameter as the new one in Fig. 11, represented by dotted outline *A A*. Our tooth space, however, as before explained, will be too narrow by the amount *m* in Fig. 11 or *p* in Fig. 12. To widen it out sufficiently, it is therefore necessary for us, after the hob has been fed in to the proper depth, to still continue the cutting action, feeding the hob endwise, however, until it has been displaced to the position indicated by outlines *D' D'*. The resulting tooth is evidently identical with that given by the new hob *A A* in Fig. 11.

It will be understood that when the hob in Fig. 12 is new, it will not have to be shifted end-wise at all, since it will cut a tooth space of the proper width as soon as fed to depth. It will, however, cut a space deeper than necessary by an amount *o*. The worn hob, on the other hand, has to be shifted longitudinally by an amount *p* and cuts to exactly the required depth. These represent the two extreme con-

ditions. When the hob is half worn, the excess clearance will be equal to half of *o*, and the longitudinal displacement necessary will be equal to half of *p*.

While the change in the design of the hob could be made easily enough, there is doubtless some difficulty in making the required change in the hobbing of the blank. Taking it for granted that the hob has been made to suit the worm which is to be used, and that it, therefore, has the same pitch diameter and thickness of tooth at the pitch line, the method of procedure will invariably require that the hob be fed in to the worm-wheel blank until the distance from the center of the hob to that of the wheel is the same as the distance from the center of the worm to that of the wheel in the finished machine. This will be true whether the hob is new or worn, and whatever may be the kind of machine on which the hobbing is done.

The method by which the hob is displaced longitudinally will depend on the machine used for the operation. There will be no possible way of doing it if the wheel is being finished while running loosely on centers, as is common practice when the blank has first been gashed. It is required that the hob and blank be positively geared together. If a positively driven hobbing attachment in the milling machine is being used, the matter is simple. If the hob is being driven by the spindle of the machine, throw in the cross feed in either direction until the required longitudinal displacement of the wheel with relation to the hob has taken place. The question as to when this has taken place may be decided either by measuring the thickness of the tooth, as in cutting spur gears, or by trying the wheel from time to time with its worm, the two parts being mounted in place in the machine they are to go in, or held the proper distance apart by other means.

For regular hobbing machines, as at present made, the matter is more difficult. The required longitudinal displacement of the hob may be obtained, in effect, by a rotary displacement of the hob which may be accomplished by slipping (a tooth at a time), the teeth of the change gears connecting the hob and the blank. If a hobbing machine were to be built especially for use in the way which is here suggested, differential gearing could be introduced in the train between the hob and the wheel, to which a power feed could be given to effect the rotary displacement when the hob has been fed to depth, or a power feed might be applied to feed the spindle and its attached hob endwise to effect the same result.

It is not certain that the error which exists in the use of relieved hobs is of enough importance to warrant taking any trouble to remedy it. It is always well, however, to know and understand such errors as may exist in any process of this sort, no matter if they are of no great practical importance. While some designers and shop men have doubtless recognized the existence of this particular error, still probably most of them take it for granted that the process is absolutely accurate, since they are so often reminded that the relieved hob can be "ground without change of shape."

CHAPTER IV

THE LOCATION OF THE PITCH CIRCLE IN WORM GEARING

Different authorities and writers on mechanical subjects have always held very different opinions regarding the location of the pitch circle of a worm gear. No better example of these differences in opinion can be given than by repeating a discussion in relation to this interesting subject which took place in the columns of MACHINERY, during 1905. The subject was brought up by Mr. Oscar E. Perrigo, who, in describing the feed arrangement of a heavy turret lathe, into the design of which the worm and worm-gear entered, found occasion to state his opinions in regard to the construction of this mechanism. Mr. Perrigo says*:

"Many good mechanics are so prone to object to any kind of a worm-gear, and can cite numerous examples wherein they have proven failures and utterly worthless for the purposes intended, that there is a very strong prejudice against them in any form. The writer is of the opinion that there is really only one practical objection to a properly constructed worm-gear, and that is, it must be constantly lubricated, and men running machines in which they are used are very liable to forget this fact altogether. The principal, and almost the only reason why worm-gears fail to give satisfactory results is that usually they are not properly designed at first. Another is that they are not properly hobbed out, and sometimes not hobbed at all. It is the purpose of this article to point out how they should be designed in order that they may be successful.

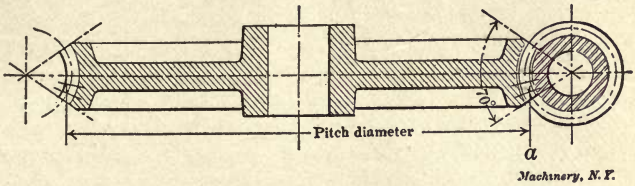
"There are various methods for determining the diameter of the pitch circle of a worm-gear. One authority takes the outside diameter of the turned blank at its smallest diameter, or throat, as proper. Another takes the diameter of the bottom of the teeth at the extreme edge of the cut gear; still another, the point where the pitch line of the worm intersects the center line passing through the worm and worm-gear. All these are more or less in error, as they do not take proper account of the width of the face of the gear. If the teeth are straight, as in a spur gear, we naturally take a point in the center of the teeth (after subtracting the clearance) as the pitch line. Now when we have a curved tooth it obviously is not proper to do this, as the actual working pitch diameter must be somewhat larger than this; but how much larger should evidently be determined by the amount of contact with the worm, that is, the angle within which this contact is to be, the width of face being in turn controlled by the diameter of the worm.

"Practically, the face of the worm gear is about equal to one-half

* MACHINERY, June, 1905.

the outside diameter of the worm, but the matter is best considered by saying that the enclosed angle of contact should not be less than 45 degrees nor more than 80 degrees, while from 60 degrees to 70 degrees will be found most useful. The writer has found by ample practice that the true working pitch diameter is most nearly determined by the method shown in Fig. 13, which represents a worm-wheel having a contact of 70 degrees. To determine the pitch diameter, divide the arc of the pitch line of the worm, contained between the center line and one of the lines of the enclosing angle, into three equal parts, and draw the line *a* at the intersection of the second line from the center line. This will give the point from which to measure the pitch diameter. If this is laid out on a large scale and with various angles of contact, the difference between it and the usual methods will be more clearly shown than it is in the engraving." It will be found to make a difference of several teeth in a worm-wheel of a fairly large number of teeth.

As to the proof of the correctness of this method of designing worm-gears, Mr. Perrigo states that he has used it successfully for years.



Machinery, N. Y.

Fig. 13. Method of Determining the Pitch Diameter of a Worm-gear

The turret lathe, previously referred to, on which this worm gearing acted as a drive for the feed, would readily bore 3-inch holes in 50-point carbon steel spindles. In several cases where a 5⅛-inch hole was required, it was first bored 2 inches and then a boring bar, provided with two double-end cutters, was introduced, enlarging the hole from 2 inches to 5⅛ inches at one cut and taking out nearly thirty pounds of chips per hour. The machine had been in use for over seven years, and the same worms and worm-gears were on it that were put on when the machine was first built, and they were in good condition for as many years more of good service. The working faces did not seem to have changed their original form during the entire time, which, Mr. Perrigo says, may be taken as ample evidence that they were right originally, particularly as he had frequently seen worm-gears in lathe aprons, designed after the usual methods, entirely worn out with six or eight months' service.

Undoubtedly prompted by Mr. Perrigo's statements in regard to the worm-gear, Mr. John Edgar, a few months later,* added to the discussion on the subject. He mentions first the method for the location of the pitch circle accepted as standard practice. According to this method the pitch line of a worm is located on a circle whose radius is smaller than that of the worm by an amount equal to one-half the

* MACHINERY, October, 1905.

working depth of the tooth. Where the working depth, as in standard practice, is equal to 0.6366 times the linear pitch, and when P' is the linear pitch, o the outside diameter, and d the pitch diameter of the worm, this fact may be expressed by the following formula:

$$d = o - 0.6366\ P' \qquad (1)$$

In Fig. 14 we have a section through a worm and worm-gear. The pitch circle for the worm, according to standard practice, is located as shown tangent to the line E, which is the pitch line of the worm-

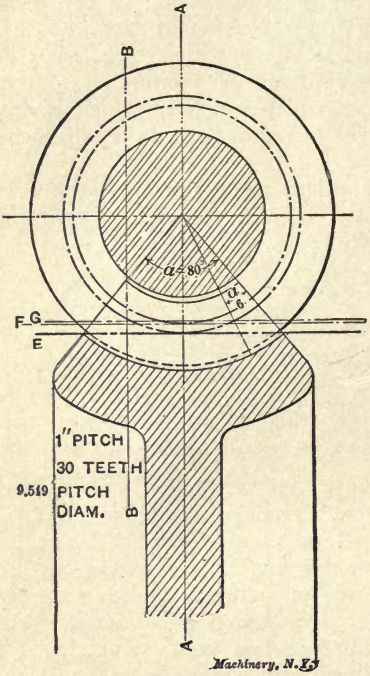


Fig. 14

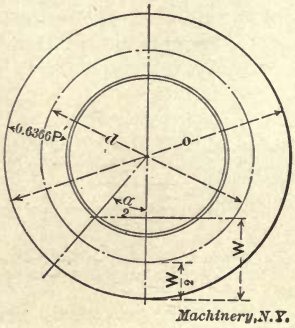


Fig. 15

gear. On inspection of the figure it is seen that while the addendum of the worm and worm-gear are equal at the center line AA, they are not at any other point along the pitch line, either to the right or the left. A section taken through the gear on the line AA would reveal teeth similar in shape to those of a spur gear of the same pitch and number of teeth. But how does this shape of the teeth vary as we shift this line either side of the central position? Let us show this by an example, taking the case of a worm having a single thread of 1-inch pitch. By taking a section on line BB instead of the center line AA we obtain Fig. 16. This figure shows plainly that the faces of the teeth of the gear are considerably longer than the flanks. It is easily seen that the greater the angle α is, the greater will this difference be, and *vice versa*, until we reach the central position, where there is

no difference. Therefore we see that this angle a plays an important part in the design of a successful worm-gear.

This angle is not the only cause of distortion in the shape of the tooth. With a little thought it will be seen that the angle of the helix also is a cause for further irregularity. To illustrate this we will take

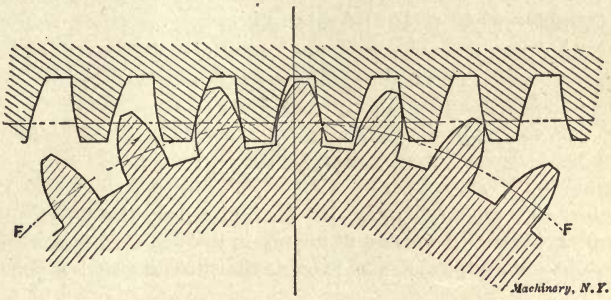


Fig. 16. Section at Line BB, Fig. 14, Single-thread Worm, One-inch Lead

the case of a worm having the same pitch, but having three threads instead of one, giving a lead of 3 inches. A section of this at *BB* is shown in Fig. 17. These conditions have the effect of producing even longer faces than do those in the former case.

What can be done to remedy this defect? We can shorten the faces, but when we do that at this point we do so all along the face of the gear and thus change the shape at *AA*, where it is normal. Therefore, the best we can do is to divide the difference at the two extreme points—*AA* and *BB*. This can be done as follows: In an ordinary spur gear of standard proportions the pitch line is located at a point midway of the working depth. From Fig. 15, which shows the end

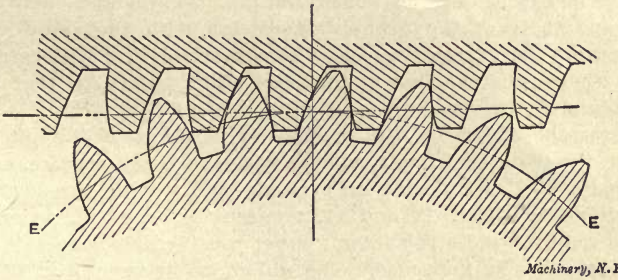


Fig. 17. Section at Line BB, Fig. 14, Triple-thread Worm, Three-inch Lead

view of a worm, we see that the total working depth is equal to W, so that from the foregoing statement the pitch line should pass through a pointed situated at a distance equal to one-half of W from the outside of the worm, making d the pitch diameter of the worm.

By an inspection of Fig. 15 we may derive the following formula:

$$W = \frac{o}{2} - \cos\frac{a}{2}\left(\frac{o}{2} - 0.6366P'\right) \qquad (2)$$

Since $d = o - W$, we may obtain the value of d in terms of o, P' and a:

$$d = \frac{o}{2} + \cos\frac{a}{2}\left(\frac{o}{2} - 0.6366\,P'\right) \qquad (3)$$

Solving this last equation for o, we have the means for finding the outside diameter when d, P' and a are given:

$$o = \frac{2\,d + 1.273\,P'\cos\frac{a}{2}}{1 + \cos\frac{a}{2}} \qquad (4)$$

Formulas (3) and (4) may be for obtaining the pitch diameter of any worm when the outside diameter is known, and *vice versa.*

It is quite evident, says Mr. Edgar, that the method given by Mr.

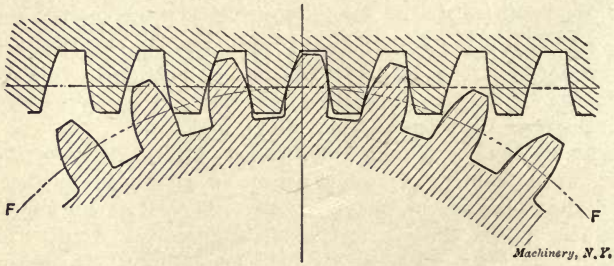


Fig. 18. Section at Line BB, Fig. 14, Pitch Line Determined by Formula (3)

Perrigo for obtaining the pitch diameter of the gear is based on this principle, but it is only an approximation, the variance between its results and those of the formula increasing with the angle a. The difference for the example we have been investigating will be seen in Fig. 14 where G is the line as located by his method, F that by the formula, and E the standard location.

To show the difference this change in location of the pitch line makes in the tooth shape as compared with the usual practice, sections have been drawn at *BB* for both a single- and a triple-threaded worm of 1-inch pitch. Figs. 18 and 19, respectively, show these sections. Here we see that while the faces are yet considerably longer than the flanks, the shape is improved. The difference between Fig. 18 and a normal section is very slight and hardly noticeable, and while the shape in Fig. 19 is somewhat freakish, it has all the properties of a smoothly running gear.

But someone may ask what all this has to do with the durability of the gear. It is this: It has been proved that the friction of approach is much more in amount than that of the release. This friction of approach occurs between the face of the driven gear and the flank of the driver. Now if these particular elements of the tooth are extra long, the friction is proportionately increased over what it

would be in a normal tooth. The friction of motion is always accompanied by wearing of the surfaces in contact; therefore in order to increase the life of the gear, we must decrease the friction to a minimum. This we have done by locating the pitch line in accordance with the formula.

In order to illustrate the extent to which some designers go to eliminate the friction between the surfaces of the teeth in contact, the case of some special forms of clock gearing may be cited where the driver is made with teeth having no flanks and the driven gear with teeth having no faces, fixing all the contact at the period of release. The importance of this point is easily ascertained by observing the wear on the teeth of a pair of gears that run constantly in one direction.

The tooth curves in the above figures were obtained by the tracing cloth method described in Unwin's "Machine Design." The subject in

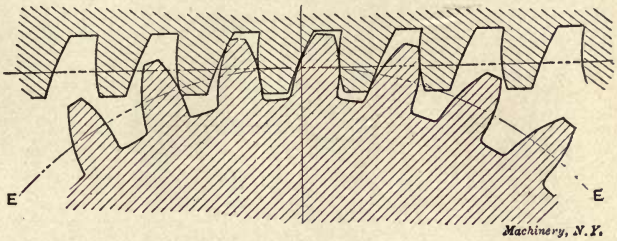


Fig. 19. Section at Line BB, Fig. 14, Pitch Line Determined by Formula (3)

hand, however, does not require or warrant the description of this method here.

Finally, Mr. Ralph E. Flanders added to the discussion by a more fundamental study into the principles involved than had been undertaken by any of the previous writers. His analyzation of the subject clears some of the doubtful points at issue. In order to give a comprehensive idea of his statements, his treatment of the question has been given verbatim in the following*:

On the Location of the Pitch Circle in Worm Gearing

Mr. Perrigo and Mr. Edgar, in their recent contributions on this subject, have called attention to some important points in connection with this form of gearing. The writer feels, however, that the recommendations they make cannot be followed blindly, but must be applied with a full knowledge of the limitations within which these recommendations are useful. It is the purpose of the present article to point out these limitations.

Mr. Perrigo describes a worm and a worm-wheel which he has incorporated in the feed mechanism of a screw machine. Made in the way he describes, this worm and wheel have outlasted everything of their kind in his previous experience, and if the cases with which he mentally compares this one have no other important points of difference,

* MACHINERY, November, 1905.

his confidence is certainly justified. Unfortunately, this point is not covered, and so we are left without a solid foundation on which to base our judgment.

The feed worm of a screw machine, if it is of the class in which the worm is dropped out of engagement when the feed is released, does its work under peculiarly trying circumstances. The writer's experience in screw machine design has led him to believe that the proper proportioning of these parts is a matter of considerable importance. Consider the case of a bronze wheel and a hardened steel worm working under the pressure of a heavy cut: When the worm is released from engagement with the wheel, under the pressure of this heavy cut, the sharp, hardened corner of the worm-tooth goes sliding down the face of its corresponding tooth in the wheel, giving it a last dig as it jumps by the corner. The necessity for quick handling

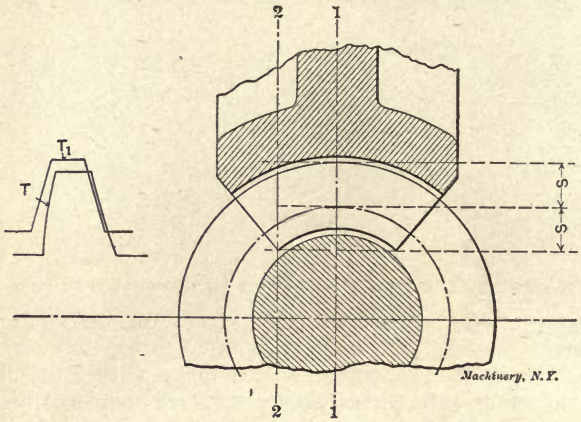


Fig. 20

demands that the momentum of the revolving parts of the feed mechanism be kept as low as possible, so the peripheral speed of the worm-wheel must be as low as possible in comparison with the rate of movement of the slide. This, in turn, requires the worm to work under heavy pressure. It is not practicable to locate the feed release between the worm and the clutch, especially if the feed is to be stopped automatically, because it is difficult to handle a toothed clutch under a severe torsional strain. Usually this problem is settled by a compromise, the success of which depends on the designer's judgment; the peripheral speed of the worm-wheel is made as high, and consequently, the worm thrust is made as low as is possible without too great a sacrifice in rapidity of handling. In large machines this difficulty may be overcome by connecting the pinion shaft to the worm-wheel by frictional contact, accomplished by tightening up a supplementary pilot mounted in front of the main pilot wheel; the automatic release is effected by stopping the rotation of the worm.

Another point that militates against the durability of this mechan-

ism when a releasing worm is used, is the indeterminate location of the worm. While it is obvious that a worm cannot be adjusted in a direction parallel to the axis of the worm-wheel, it is not generally realized that the center distance between its axis and that of the wheel cannot be varied without losing the perfect action which exists when the worm is properly located. That this is so will be evident from Fig. 20. In this cut T_1 and T are sections of a worm tooth taken on lines 1-1 and 2-2 respectively. The section on 1-1 is evidently that of an involute rack tooth and so possesses the characteristic property of correct action at any center distance, so long as its straight face is in contact with the mating gear tooth. As we leave this section, however, and approach section 2-2, the tooth outline gradually loses its resemblance to the involute form and takes a shape in which positive location is absolutely necessary for correct action, as is shown by the curved sides. This variation from the

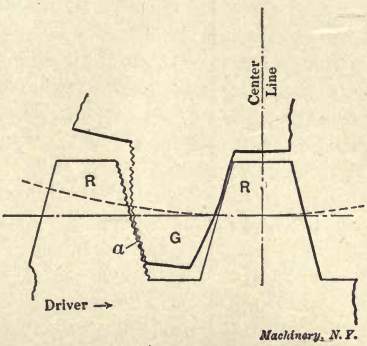


Fig. 21

involute shape is especially marked in worms of large helix angle and consequent high efficiency.

Now, if the worm is slightly separated from its correct location in the mating wheel and no sideways motion is allowed, it will be seen by observing the relative angularity of the outlines of the faces in the curves T and T_1 that the contact will at once lose its character of line contact, extending across the full width of the gear, and will be concentrated in point contact on the extreme outer edge, where correct action is impossible except at the calculated center distance. For working under heavy pressure, then, it is necessary that the worm agree in shape with the hob which cut its mate, and that its axis exactly coincide with that of the hob when this was taking its finishing cut. These requirements may be met easily in high-grade work, such as is the rule in making a worm-gear drive for a gear-cutter spindle or an elevator, but such workmanship is very far from the haphazard fitting that a releasing feed worm must necessarily get.

It has occurred to the writer that the worm, or worms, in Mr. Perrigo's turret lathe, must be of considerably greater helix angle than is usual in feed gearing. The unusual arrangement of a double reduc-

tion is employed, making use of two sets of worms and wheels in series. Unless the feed shaft rotates at high speed, or the feed is exceedingly fine, this must mean that the reduction in each set of gears is small, which in turn predicates a large helix angle and an efficient gear. Mr. Perrigo must, then, give us more definite information if his experience is to be valuable as a permanent record in the matter of the location of the pitch line. Was his machine furnished with a releasing worm for a feed stop, and were the machines with which he compares it so equipped? How carefully was the worm

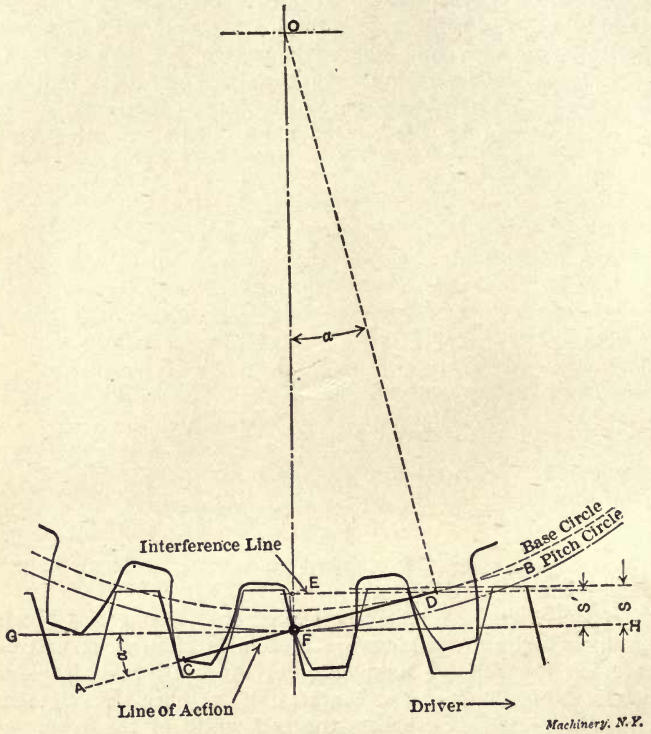


Fig. 22

fitted in the last machine and in the former machines? What are the helix angles of these worms and former unsuccessful cases? What materials were used in the different sets of gears which are under comparison?

Mr. Edgar has shown quite plainly that the advantage to be gained by lessening the diameter of the pitch circle on the worm is due to the fact that in such a case the contact between worm and wheel takes place for the most part after the teeth have begun to recede from each other. In Fig. 22 the worm, with its pitch line at *GH*, driving the wheel in the direction shown, will always make contact with it along the line of action, *CD*. The pitch line is located, as usual, half-

way down the working depth of the tooth, and as may be seen, the contact is almost equally divided on each side of the center line. In Fig. 23, with the same reference letters, the pitch line has been located according to the rule proposed by Mr. Edgar, and the contact between the teeth is seen to take place almost wholly during the time when they are leaving each other.

Friction between two rubbing surfaces is due to the resistance imposed by the microscopic irregularities which exist on even the smoothest surfaces. In Fig. 21 are shown two teeth approaching each other, in which these irregularies are greatly exaggerated. *R* is the driving and *G* is the driven tooth. Evidently if these irregularities were as great as shown, the teeth would lock together and movement would be impossible; on the other hand, if *G* were the driving tooth, and the teeth were separating, there would be little to hinder their free movement. It is, then, desirable that most of the contact should take place when the teeth are leaving each other, to avoid friction, loss of power, and wear of tooth surfaces.

Fig. 20 shows the way in which Mr. Edgar proposes to locate the pitch circle of the worm. This circle is tangent to a line which lies at equal vertical distances from the extreme working points of the worm-wheel tooth, and he locates the pitch line here because it is so located in a spur gear. To the writer it seems that there is no analogy between them. The pitch line of a spur gear is located at one-half the working depth of the tooth because it is required that a set of standard spur gears be interchangeable, a gear of any number of teeth meshing perfectly with a gear of any other number of the same pitch. This requirement is entirely outside of the sphere of worm gearing, so we may locate the pitch line at any point that will give favorable results as regards efficiency and durability.

The location of the pitch line affects the working qualities of the gearing in four ways, at least. With a worm of given diameter and pitch, and a wheel of given number of teeth and angle of contact, it determines the effective working area of the teeth in both members, the strength of the teeth in the wheel, the number of teeth in contact, and the nature of the contact, that is, whether it takes place during the approach or the release.

Fig. 22 shows a central section of a worm and wheel calculated in the usual manner. If α is equal to the pressure angle, and angle *FDO* is a right angle, a circle drawn from center *O* through *D* will be the base circle from which the involute curve is formed, and the line of action—the line in which the working contact between the teeth will take place—will lie in line *AB*. This line of action will evidently be limited at one end by *C*, the point where it crosses the outside diameter of the wheel at its throat, and at the other by *D*, the point where line *AB* is tangent to the base circle, since the involute does not extend inside of the circle from which it is derived. It is plain, then, that all that part of the wheel tooth which lies inside of the base circle is clearance, and unfit for bearing surface, and that all of the worm tooth which extends above point *D*, or the "interference line," as it is

marked, serves no useful purpose. This area of the worm tooth extending above the interference line, is seen to be slight for a thirty-tooth wheel of standard design. An inspection of the cut will show that there are always two and sometimes three teeth in contact. The contact takes place about equally each side of the center line, inclining toward the favorable side, since line *FD*, on the release, is somewhat longer than *CF*, on the approach.

In Fig. 23, we may see what effect has been produced by increasing the addendum of the worm, as we are advised to do. In the first place,

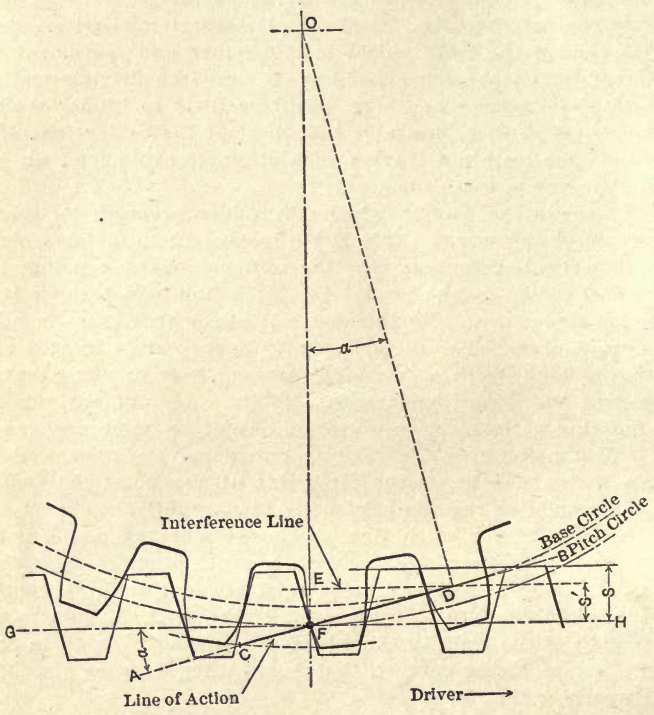


Fig. 23

the interference line, through point *D*, is lowered so far that the useful area of the teeth has been greatly decreased. There is no working contact on the wheel teeth inside of the base circle, nor do the worm teeth serve any useful purpose above the interference line.

The strength of the wheel tooth has been decreased. The great length of useless worm tooth extending above the interference line has cut a deeper clearance into the flank of the wheel tooth, thus weakening it at the very point where it needs strength. On pages 68, 69, and 71, of the Brown & Sharpe "Treatise on Gears," are three illustrations which show this point very clearly. One case is that of a twelve-tooth worm-wheel of standard design, which gives a badly

undercut flank. In the next illustration the worm-wheel has been hobbed according to Mr. Edgar's rule, and the result is worse than in the first case. The last illustration shows the pitch line thrown clear to the outside diameter of the worm, this being advised as the proper remedy to secure a tooth of sufficient strength and bearing surface.

Referring again to Fig. 23, the number of teeth in contact has been reduced until there is only one constantly in use, though two are in position to work most of the time. The single gain to be derived in return for the advantages that have been lost, lies in the fact that a greater percentage of the line of action lies on the releasing side of pitch point *F* than before, since *FD* is noticeably longer than *FC*.

Of course only the action on the center line has been analyzed. The writer has studied the action at sections made in different places in the worm-wheel face, and it looks as if the conditions at the center line were a fairly good index of what is going on nearer the sides. The line of contact appears to rise slightly toward the outside of the worm as it leaves the center (going toward the leading side of the worm), and then drops again toward the edge of the wheel. On the retreating side of the worm the contact drops continuously. This tends to minimize the effect that the width of the wheel has on the action.

How, then, should the pitch line be located? It seems to the writer that the problem is so involved that in a case of any importance the designer should not trust to any empirical rule, but should plan each case with reference to these four points: area of bearing surface in the teeth, strength of the teeth, number of teeth in contact, and location of contact, whether in the approach or the release. To these should be added a fifth point, more important than any of the others, as far as efficiency is concerned, and that is in relation to the helix angle of the worm: it should be as large as possible.

Taking all these points into consideration, it would seem that, for worms and wheels made as they usually are for ordinary service, from ordinary materials, and with ordinary carefulness of workmanship in making and fitting, it is hardly worth while to bother about changing the location of the pitch line for the sake of having the contact on the release. It introduces too many other complications into the problem. Still, if there is any one who wants to try the effect of altering the worm and wheel dimensions with this end in view, here are a few suggestions in the shape of formulas to add to those of the two contributors who have previously written on this subject.

Let N = number of teeth in wheel.
O = throat diameter of wheel.
P' = linear pitch of worm.
o = outside diameter of worm.
D = pitch diameter of wheel.
d = pitch diameter of worm.
a = pressure angle.

$$C = \frac{D + d}{2} = \text{center distance between the worm and the wheel.}$$

S' = effective height of worm tooth above pitch line (see Fig. 23).

An inspection of Fig. 23 will show that S' may be expressed as follows:

$$S' = \frac{D \sin^2 a}{2}$$

If we limit the height of our tooth to this line, thus allowing no interference, we may use the following formulas, it being considered that we have given C, P' and N.

$$D = \frac{N P'}{\pi} \tag{5}$$

$$d = 2C - D \tag{6}$$

$$o = d + D \sin^2 a \tag{7}$$

$$O = D + 1.273\,P' - D \sin^2 a \tag{8}$$

For a pressure angle of 14½ degrees and an allowed interference equal to that of a standard worm in mesh with a 25-tooth wheel, these last two formulas will become:

$$o = d + \frac{D}{13} \tag{9}$$

$$O = 0.923\,D + 1.273\,P' \tag{10}$$

These formulas will give as much of the contact on the release as is possible without too much undercutting; the location of the pitch line will, of course, vary widely. Formulas (7) and (8) (when a = 14½ degrees) are good for any number up to 64 teeth, and Formulas (9) and (10) up to 52 teeth. Above these numbers the formulas would bring the pitch line below the root diameter of the worm, which is needless; so for such cases, Formulas (7), (8), (9), and (10) should be replaced by the following, which will keep the pitch line within the working area of the tooth:

$$o = d + 1{,}273\,P' \tag{11}$$

$$O = D \tag{12}$$

All that has been said in the preceding paragraphs refers only to worms whose tooth outlines show straight sides on an axial section. If, as is often the case with steep-pitched worms, the cutting tool is made with straight sides, but tipped up at an angle to agree with the helical angle of the worm, an axial section will show teeth with curved sides whose shape will depend upon the helical angle. In such a case as this it is impossible to apply any of the rules which govern the action of involute teeth, and the only way to go about the matter of locating the pitch line to suit the ideas of the designer is to make a careful analysis of the tooth action on various sections. This operation would be so troublesome and tiresome as to be impracticable under any ordinary circumstances.

CHAPTER V

THE HINDLEY WORM AND GEAR*

The Hindley type of worm-gear was first used in Hindley's dividing engine,† and was, by the inventor, considered superior to the ordinary type, in wearing quality. Investigation has practically settled that the nature of contact between the worm thread and the teeth of the ordinary worm-wheel is that of line contact, extending across the tooth on the pitch line. It has also been fairly well proved in prac-

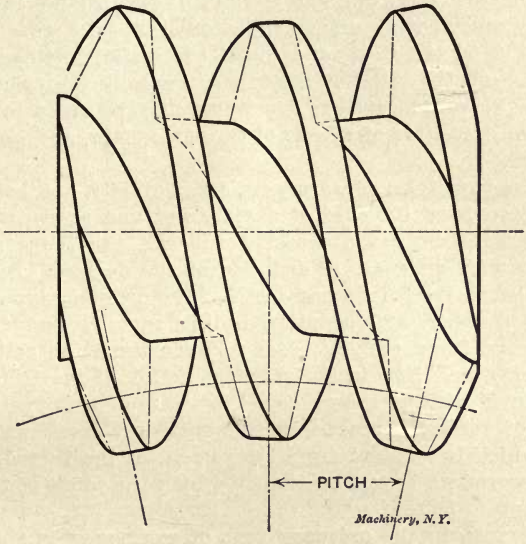


Fig. 24. Typical Hindley Worm

tical examples that the contact is of a broader nature on account of the elasticity of the materials used in the construction. The convex surfaces of contact are flattened considerably under pressure and thus for practical purposes make actual surface contact. The contact in the ordinary worm and worm-wheel type is limited to two teeth of the wheel and worm thread, at most.

Comparison of Ordinary and Hindley Worm Gearing

The conditions are much different in the case of the Hindley worm, and it is the intention in this chapter to show wherein the difference lies. As this style of gearing is uncommon to most of us, a few words

* MACHINERY, December, 1908.

† The Hindley gear, as used in the Hindley dividing engine, is described by Smeaton, also by Willis (Principles of Mechanism, 1851). Various modifications of the Hindley gear, including Jensen's winch, are illustrated in Reuleaux's "Constructor," page 143.

regarding its construction will not be out of place. Fig. 24 illustrates the Hindley worm, showing the theoretical form. This worm is not of cylindrical shape, but is formed somewhat like an hour-glass, after which it is sometimes named. The worm blank, being made smaller in diameter in the middle than at either end, conforms to the circumference of the wheel with which it meshes. The worm thread is cut by a tool which moves in a circular path about a center identical with the axis of the wheel with which it is to mesh, and in the plane in which the axis of the worm lies. The process is similar to ordinary thread cutting in the engine lathe, except for the difference in the path of the tool, the tool having a circular instead of a straight path.

It is evident that the worm shape is dependent on the particular wheel with which it is to run, and Hindley worms are not interchangeable with any other but an exact duplicate. That is, a worm cut for a Hindley gear of 50 teeth cannot be used successfully with a wheel of 70 teeth, although the pitch of the teeth is exactly the same. In the ordinary type of worm gearing, one worm may be made to run with any number of diameters of wheels of the same pitch, and hobbed with the same hob.

In action the two styles of worm-gear differ greatly, and both diverge widely in action from the case of a plain nut and screw, which may be taken to represent a worm and worm-gear, the latter of infinite diameter and with an angle of embrace of 360 degrees. In studying the action between the thread and teeth of the ordinary type of worm-gear, we must understand odontics, rolling contacts and the theory of tooth gearing, in general, in order to understand the action of the ordinary worm-gear. But, in studying the action of the Hindley type, we are concerned with no such theories, as the action is purely sliding and devoid of rolling contact. In the ordinary worm we have an axial pitch which is constant from top to root of the thread, while in the Hindley worm we have a section in which the pitch of the thread varies from top to bottom.

The interference in the ordinary type of worm-gear is absent from the Hindley type, and the consequent undercutting and weakening of the teeth, therefore, is a feature with which the designer of the Hindley worm gearing does not have to contend. For this reason we are not limited in the length of teeth, by interference, as in the ordinary case. This fact permits a wide latitude in the choice of tooth shapes and proportions. In most examples we will find that the depth of thread is much greater in proportion to the thickness than in the ordinary worm-gear, in which the height is limited by reason of the interference at the top and root of the teeth.

Nature of Contact of Hindley Worm Gearing

The general idea of the Hindley worm gearing is that there is surface contact between the worm and gear, and that the contact is generally over the whole number of teeth in mesh. If such were the actual conditions, the Hindley type would surely be an ideal mech-

anism for high velocity ratios, but that such is not the fact is the purpose of this treatise to point out. That the contact is of a superior nature we will not deny, nor that it is much nearer a surface contact than exists in the ordinary worm gear. As a means of comparison, Figs. 25 and 26 are shown. Fig. 25 shows an axial section taken through the worm and gear of the ordinary type, while Fig. 26 shows a similar section through the Hindley worm and gear. The "airy"

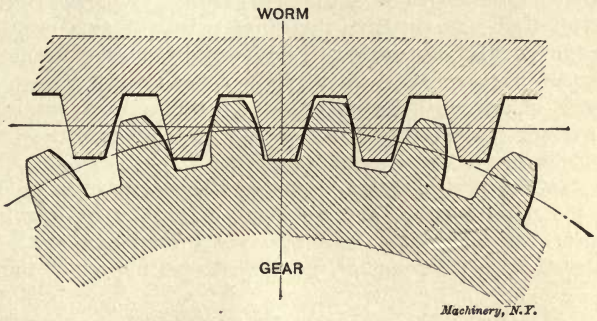


Fig. 25. Section of Common Worm and Worm-wheel on Middle Plane

appearance of Fig. 25 as compared with Fig. 26, indicates a vast difference in the nature of contact, and gives the advantage to the Hindley type, wherein is the origin of certain false ideas in favor of the latter. These illustrations also show peculiar differences in the action of the two types. The absence of rolling action in Fig. 26 is the most prominent, and it shows the similarity between this type of gear and a screw and nut.

From an inspection of Fig. 26 we may feel sure that the contact on

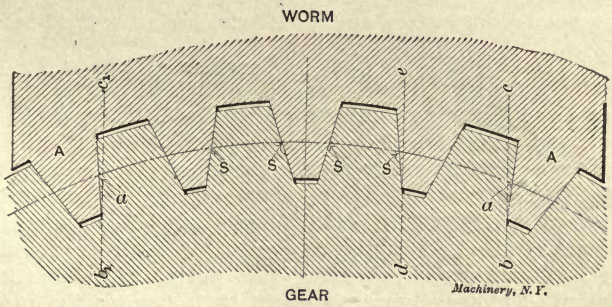


Fig. 26. Section of Hindley Worm and Gear on Middle Plane

the axial plane is as shown, but as to the nature of contact in a plane either side of the middle plane we are in the dark so far as the drawing illustrates. Mr. George P. Grant has this to say concerning the contact of the Hindley worm and gear: "It is commonly but erroneously stated that the worm (Hindley) fits and fills its gear on the axial section. . . . It has even been stated that the contact is between surfaces, the worm filling the whole gear tooth. . . . It is

also certain that it (the contact) is on the normal and not on the axial section, and that the Hindley hob will not cut a tooth that will fill any section of it. The contact may be linear on some line of no great length, but it is probably a point contact on the normal section."

It is not clear what reason Mr. Grant had for saying that the contact is normal instead of axial; but there is every reason to believe that the contact is on the axial section since it is on this section that the teeth of the hob have a common pitch. The teeth have not a common pitch on any section at an angle with the axial section. For what reason would one expect to find contact on the normal section in this case any more than in the case of the ordinary worm? Since both styles of worm-wheels are hobbed with a revolving hob which lies in a plane perpendicular to the axis of the worm-wheel, the contact could hardly be on a normal section.

Prof. MacCord states that he considers the contact to be line contact on the axial section, and he gives directions for obtaining the exact nature of the contact and also the thread and tooth sections. These directions, on account of the complicated nature of the method,

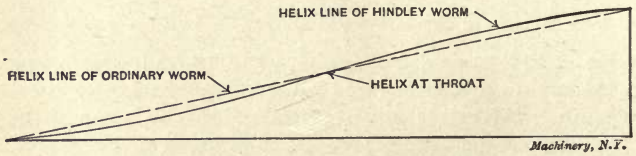


Fig. 27. Development of Ordinary Worm and Hindley Worm Spirals on a Plane

are hard to follow. Much, however, can be found out by simple methods. In what follows, describing these simpler methods, the results, of course, are of an approximate order, but they nevertheless give a means of comparison and a material basis for the line of argument.

The Ideal Case Considered

It is assumed that we are examining an ideal Hindley gear in which the worm and wheel are theoretically correct in shape and that the surfaces are perfectly smooth and inelastic. From the nature of the worm, the helix angle varies from mid-section to the ends, decreasing as the thread approaches the ends of the worm. The thread is spiral as well as helical. This change in the thread angle is caused by the increase in diameter at the ends of the worm and by the fact that the axial pitch of the thread decreases as it reaches the ends. The decrease in axial pitch is due, of course, to the circular path of the threading tool. If we take a development on a flat surface of a line scribed in the spiral path on the worm blank, as shown in Fig. 27, the change in the angle becomes noticeable.

In the operation of forming the teeth of the gear, the blank is rotated, each portion of the hob working the tooth into shape so that it will pass the corresponding portion of the worm thread without interference, permitting a smooth transmission of motion. If each

portion has a different shape or is placed in a different relation, the shape of a gear tooth will be a compromise between the extremes, and this is what is actually the result, as we shall see later.

The progressive steps of the process are shown in Fig. 28; the successive positions of one tooth are shown, beginning at the left and ending at the right-hand position where each tooth is given its final shape. The nature of the process is shown in Fig. 29, the shaded portions representing the gear teeth. Here we have a representation of the contact of the thread and teeth; it shows that surface contact is impossible on any but the heavily shaded portions of the teeth, it being confined to the mid-section and the extreme end sections of the worm. Line contact is obtained throughout the length of the worm on the axial plane. This figure also shows that no advantage is gained in surface contact by making the worm of greater length. The location of the contacts are shown in Fig. 29, at *a*, *s*, *s*, *s*, *s*, *a*, but it

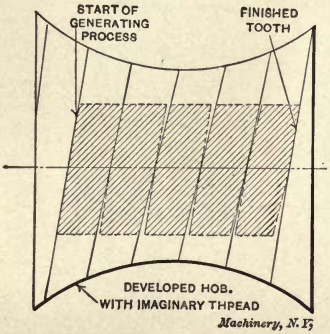


Fig. 28. Successive Steps in Shaping the Hindley Worm

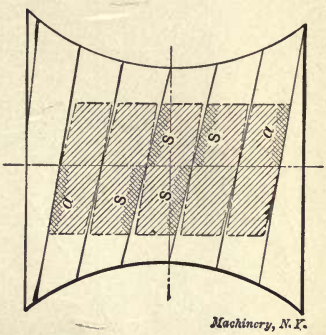


Fig. 29. Surfaces of Contact of the Hindley Worm

must be remembered that they lie on opposite sides of the cutting plane. From this it is apparent that the worm does not entirely fill the space between the teeth of the gear and that the contact is not wholly a surface contact.

Let us investigate still further and see whether the conditions are not modified by other irregularities: Fig. 30 is drawn to represent a worm and gear of the Hindley type, in mesh, the teeth of which have no depth. As before mentioned, the peculiarity of this type of worm is its hour-glass shape. The hob and worm may be treated as identical in form. In the process of generation, the tooth has a pitch line curvature that changes with corresponding positions in relation to the thread portion acting upon it. The tooth must necessarily be modified from what it should be for any particular location in its contact with the worm thread. It is quite clearly shown that if the tooth is to fill the worm thread or *vice versa*, it must be formed in strict accordance with the thread at that particular point. Thus if at *j* the tooth fills the thread, that tooth must be formed by the thread at that point, while the tooth at *k* must be formed by the thread at *k*. Now, since each tooth must pass from *k* to *j*, its form must be such

that it will do so without interference. It is evident that the radial section of the gear at *k* must be the same as at *j*. Since the worm is largest in diameter at *k*, the curvature of the tooth on the radial section is dependent on the thread at that point. The curvature of the tooth at *k* evidently is that of an ellipse whose major axis is $A A_1$. Now, since the thread is made with angular sides, the hob could hardly act on the teeth of the gear the same at all points from *k* to *j* except on the axial plane where the relative shape of the hob thread is the same for any position along the line of action (see Fig. 26). This is evident from Fig. 30 at *E*, which point only touches at the mid-section of the worm. Therefore we still have the line contact from top to bottom of teeth on the axial plane, but the construction, Fig.

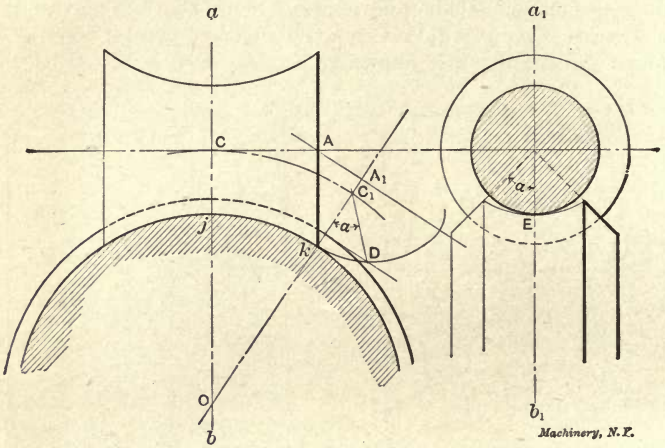


Fig. 30. Effect of Hour-glass Shape on Worm-wheel Contact

30, shows that the surface contact *s*, *s*, *s*, *s*, Figs. 26 and 29, does not actually exist, but that the surface contact at the ends of the worm remains undisturbed.

From the above we may safely conclude that the hob at *j* has but little effect on the actual shape of the tooth, and that its influence increases until *k* is reached. Fig. 30 also shows a good reason why the contact may be considered axial instead of normal, by the mere fact of the differences in curvature of worm and wheel at any point other than *k*. In practice the contact may appear to be surface contact, but this, no doubt, is due to the influence of the lubricating oil and the fact that materials of construction are distorted to some extent in form when subjected to pressure. This distortion permits the worm thread to imbed itself into the worm-wheel teeth, somewhat broadening the contact for the time being. The conditions as stated in the above discussion would be met in the case of a hardened worm and gear with surfaces finished by lapping. In practice the worm and gear are ground together, sand and water being used as the abrasive. This grinding wears down the roughness of the surfaces and tends to correct irregularities in form that develop in the hobbing process.

Objections to the Hindley Gear

The objections to the Hindley type of worm-gear are many and are widely known. It must be set up accurately, the alignment being made perfect. End play is a feature that must be avoided, as any longitudinal displacement of the worm will cause the gear to cut. These peculiarities are the greatest drawbacks to the use of this gear, and because of them the writer believes that it will not come into common use, at least not so common as the worm drive of the ordinary type. This opinion is strengthened by the fact that we have become so much more familiar with the latter type as to be able to design and construct drives that work satisfactorily in every respect.

Modifications of Hindley Worm-Gear Practice

Some modifications have been made in the process of manufacturing the Hindley worm-gear. One that is probably of first importance is that known as the "second cut," this practice being generally cred-

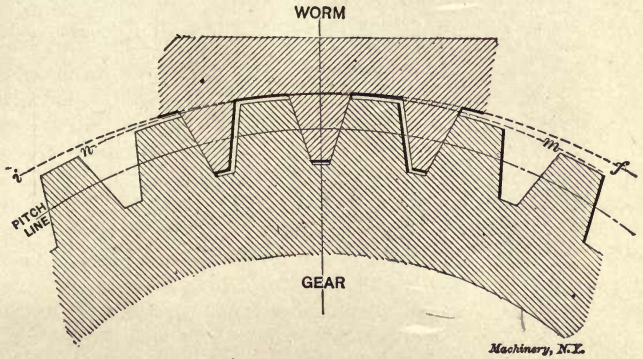


Fig. 31. Effect of the "Second Cut" on Contact

ited to Mr. Albro of Philadelphia, but the credit for it is in dispute. The effect of the second cut is indicated in Fig. 31. From this illustration one would say that the object of the second cut is to remove the points of contact. Whether this is the reason or not, it is a fact that it does remove considerable of the contact from all but the mid-section of the worm. This second cut is made by enlarging the diameter of the circle in which the threading tool travels when cutting the worm. It is said to have advantages that add to the wearing quality of the drive, but just what these advantages are is not apparent, and since the process is considered more or less a trade secret, it is difficult to obtain authentic reasons for its use.

The limiting length of the worm is dependent on the shape of the thread. In Fig. 31 the worm is shown with three teeth in mesh, while Fig. 26 shows five. Fig. 26 shows a case that would be impossible in practice on account of the undercut teeth *A* which lock the worm in mesh. The side of the thread must fall inside the line *b c* to permit the worm and gear to be assembled.

Conclusions Regarding the Hindley Worm and Gear

The following are the author's conclusions, derived from the investigation regarding the Hindley type of gear:

1. The contact is purely sliding contact.
2. The nature of the contact is linear, closely resembling surface contact.
3. Linear contact extends from the top to the root of the tooth.
4. The contact is on the axial section.
5. The thread section fills the tooth space on the axial section only.
6. The mid-portion of the hob has little or no effect in shaping the teeth of the gear.
7. Surface contact exists on opposite sides of the axial plane at the end of the worm thread and is intermittent in nature, because the end of the thread passes out of contact with the tooth in the revolving of the worm. This contact is on a plane normal with the thread angle.

In practice it is usual to allow considerable back-lash between the thread and the tooth of the worm and gear. This play tends to counteract bad workmanship, either in construction or erection.

CHAPTER VI

THE DESIGN OF SELF-LOCKING WORM-GEARS*

The old opinion that the friction and wear of worm-gears are necessarily very great, and that the efficiency is necessarily very low, making worm gearing an unmechanical contrivance, is not as frequently met with now as formerly. In Unwin's Machine Design it is stated that in well fitted worm gearing, of speed ratios not exceeding 60 or 80 to 1, motion will be transmitted backwards from the wheel to the worm. In Prof. Forrest R. Jones' work on machine design may be found tabulated the results of many examples from practice, some of which show an efficiency as high as 74 per cent before abrasion began, the most notable example being that of a worm running at a surface speed of 306 feet per minute under a load of 5,558 pounds, and showing an efficiency of 67 per cent, with no abrasion. The tables in Prof. Jones' work show that under light loads very high surface or rubbing speeds are allowable, running as high as 800 feet per minute. It has also been pointed out that an increase in the thread angle, in general, increases the efficiency.

There is, however, an important function of worm gearing which is not, as a rule, brought out adequately by writers on worm gearing, and which in certain classes of machinery is of the first importance; often, indeed, becoming the determining factor in deciding upon the choice of a worm-gear as the power transmitter. It is the property a worm-gear possesses, under certain conditions dependent upon its design, of being self-locking, and preventing motion backwards.

An instance where this property becomes of prime importance and accounts for the use of the worm-gear, is in crane work, where the winding drum is driven by a worm-gear so designed that, when the power is shut off, the gear will not run down or backwards under the impulse of the load, but will be self-locking, holding the load at any point.

Fig. 32 shows a single thread worm in mesh with the worm-wheel, a being the angle of the worm thread with the axis of the worm-wheel, and in order that the system may be self-locking, that is, that the worm-wheel may be unable to run the worm, the tangent of the angle a must be less than the coefficient of friction between the teeth of the worm and wheel, or as

$$\tan a = \frac{p}{\pi d}, \text{ so } \frac{p}{\pi d} < f \qquad (1)$$

in which p = the pitch; d = the pitch diameter of the worm; and f = the coefficient of friction between the worm and wheel. It is neces-

* MACHINERY, December, 1902.

sary to assume a value for f, which, if the condition of determining the use of the worm-gear is its self-locking property, should be assumed conservatively low. Unwin states under the authority of Prof. Briggs, that a well-fitted worm-gear will exhibit a lower coefficient of friction than any other kind of running machinery. Prof. Jones gives a series of values for the coefficient of friction of screw gears, one of which is a pinion of 4 inches pitch diameter, the average value being $f = 0.05$, corresponding to a rubbing velocity of 250 feet per minute. Mr. Halsey assumes $f = 0.05$, and Mr. Wilfred Lewis says that when the worm-gear is worked up to the limit of its safe strength, a rubbing velocity greater than 200 to 300 feet per minute will prove bad practice. It is in heavy machinery where worm-gears are mostly used as self-locking transmission elements, and here they are usually worked up to the safe strength of the wheel; hence it is fair to assume $f = 0.05$

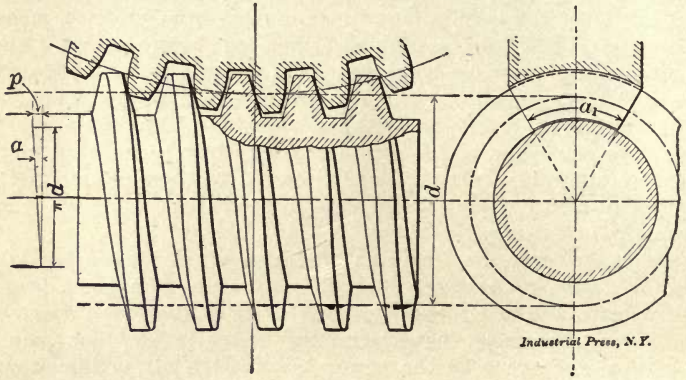


Fig. 32

when designing a self-locking worm-gear, and to limit the rubbing velocity to 200 feet per minute, and we have for the limiting value of p at which the system will be self-locking:

$$p = 0.05\,\pi\,d = 0.157\,d \qquad (2)$$

The sliding velocity in feet per minute at the pitch line is expressed by

$$V = \frac{\pi\,d\,n}{12} = 0.262\,d\,n \qquad (3)$$

where d = the pitch diameter of the worm, and n = the number of revolutions per minute of the worm.

Under the above assumption, that for continuous service and heavy pressures the sliding velocity should not be more than 200 feet per minute, we have as the limiting value of d to avoid all cutting:

$$d = \frac{200}{0.262\,n}.$$

The exact nature of the surface of contact between a worm and wheel is involved in doubt; many claim it is only a point; it certainly is not large, and consequently a wide face for the wheel is not needed

If the angle a_1 is made 60 degrees, it will make the face right for any ordinary worm of 4 to 6 inches diameter.

There is in all worm gearing a very heavy end thrust on the worm-shaft, and also an outward force normal to the worm-axis, each of which must be suitably provided for in the design of the shaft and bearings. The end thrust may be taken by bronze washers slipped into the bearings at the end of the shaft, which may be removed when worn and replaced with new ones. Shoulders may be provided on the shaft, between which and the bearings bronze collars may be placed, these being split to enable new ones to be easily and quickly placed in position when the old ones become worn. Roller thrust bearings are very often applied to worms, and these as well as the bronze washers may be supplied with adjusting set-screws to take up the wear, instead of renewing the washers.

Analysis of the Forces

In Fig. 33 let P = the tangential force at the pitch line of the worm, d = the pitch diameter of the worm, Q = the tangential force at the

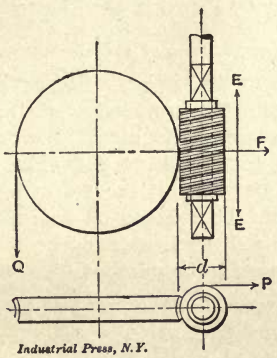


Fig. 33

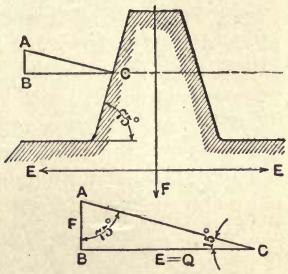


Fig. 34

pitch line of the worm-wheel, E = the end thrust of the worm-shaft, F = the force on the worm-shaft normal to the worm-axis; then, friction being neglected,

$$Q = \frac{P \pi d}{p} \qquad (4)$$

In Fig. 34, draw line BC parallel to the axis, or coinciding with the pitch line, of the worm; let this line represent the force $E = Q$; draw AB normal to this line; it will then also be normal to the axis of the worm; then, when measured to the same scale to which BC is drawn, $AB = F$; if the angle CAB is 75 degrees, we have:

$$\frac{F}{Q} = \tan 15 \text{ deg.} \qquad (5)$$

$$F = 0.268\, Q \qquad (6)$$

Taking friction into consideration, the force P_1, tangential to the pitch

line of the worm, which it is necessary to employ in order to produce a force Q tangential to the pitch line of the wheel, is given by Weisbach as

$$P_1 = Q\frac{h+f}{1-hf} \tag{7}$$

in which

$$h = \frac{p}{\pi d}$$

The efficiency of the worm and wheel is then,

$$\frac{P}{P_1} = e \tag{8}$$

Example: A single thread worm of 1-inch pitch, running 80 revolutions per minute, is to transmit to a worm-wheel a tangential force $Q = 5{,}000$ pounds, and is to be self-locking.

From (3)

$$d < \frac{200}{0.262 \times 80},$$

or d may be as large as 9.5 inches before abrasion need be feared.

From (2)

$$p < 0.157\ d;\ \text{assume } p = 0.125\ d,$$

then, as $p = 1$ inch, $d = 8$ inches, or the worm will require to be 8 inches pitch diameter in order that the angularity of the thread may be small enough to make the system self-locking. It will be seen that the required diameter will be increased as the value of f is decreased, and in case the required diameter of the worm proves too great for practice, and the pitch cannot be reduced on account of considerations of strength, some outside aid, such as a brake or friction disk applied to the worm-shaft, will have to be adopted.

From (7)

$$\text{as } h = \frac{p}{\pi d} = \frac{1}{3.14 \times 8} = 0.04, \text{ we have}$$

$$P_1 = 5{,}000\frac{0.04 + 0.05}{1 - (0.04 \times 0.05)} = 451 \text{ pounds.}$$

From (4)

$$5{,}000 = \frac{3.14 \times 8 \times P}{1}, \text{ or } P = 199 \text{ pounds.}$$

From (8)

$$\frac{P}{P_1} = \frac{199}{451} = 44 \text{ per cent for the efficiency of the worm-gear.}$$

The formulas may, by starting with those for the efficiency, be used to determine the pitch diameter which will give the proper thread angle for any given pitch and degree of efficiency.

It is clear from the foregoing, that a worm-gear of large pitch will require a pitch diameter of the worm altogether too large for practice, if it is to be self-locking, and that the system as usually designed may be expected to run backwards. To prevent this, a friction disk may

be placed in the bearing which receives the thrust of the worm-shaft when the system is running backwards, and the diameter of the disk so proportioned as to just hold the worm-shaft stationary under the impulse of the worm-wheel.

The foregoing discussion neglects the effect of the thrust of the worm-shaft in its bearings, the frictional resistance of which must be added to that of the teeth to obtain the actual conditions of a self-locking system. This frictional resistance depends upon the values of the end thrust E and the normal force F already found, and the diameter and form of the bearing. In nearly all cases of worm gearing the mounting of the worm upon the shaft will be covered by one

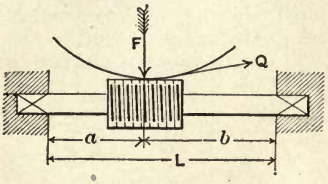


Fig. 35

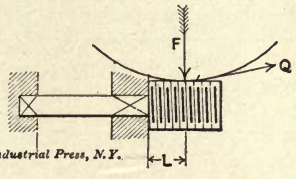


Fig. 36

of three cases, either unsymmetrically between the bearings, symmetrically between the bearings, or over-hung.

In Case 1, Fig. 35, the bending moment upon the worm shaft is,

$$M = \frac{F\,a\,b}{L} = \frac{0.259\,Q\,a\,b}{L} \qquad (9)$$

In Case 2, same as Case 1, except that the worm is central between the bearings, and

$$a = b = \frac{L}{2}$$

the bending moment upon the worm-shaft is,

$$M = \frac{0.259\,Q\,L}{4} = 0.0647\,Q\,L \qquad (10)$$

In Case 3, Fig. 36, the bending moment upon the worm-shaft is,

$$M = F\,L = 0.259\,Q\,L \qquad (11)$$

In each of the above cases the shaft is subjected to a combined twisting and bending strain, the twisting moment being the same in each case, $T = P\,R$, which is, however, so small as to be negligible in what follows.

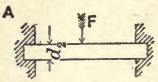
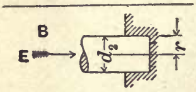
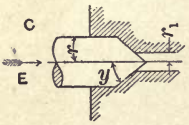
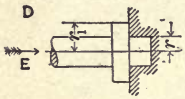
In the following table the first column shows the several styles of journals most commonly used for worm-shafts, the second column gives the moment of friction for each under a load in the direction of the arrow, the third column gives the coefficient of friction assumed, and the fourth column gives the tangential force P_2 at the pitch line of the worm, resulting from the resistance of friction in the journals, and found by dividing the moment of friction in Column 2 by the pitch radius of the worm.

There are always acting upon the worm-shaft the two forces F and

E; consequently to get the resultant retarding force tangential to the pitch line of the worm, we must take the sum of the resultants due to the frictional resistance of each force separately. Referring to the table, we will, for each worm-shaft, find the conditions shown at A, in addition to the conditions shown either at B, C or D, as the case may be, and the total resultant force P_2 at the worm pitch line will be the sum of the quantities given in Column 4 opposite the particular cases.

These frictional resistances developed by the journals act in a direction helpful to the self-locking property of the worm, and enable the designer to use a larger thread angle for a given diameter of worm,

TABLE GIVING MOMENT OF FRICTION WITH VARIOUS TYPES OF BEARINGS

Style of Journal.	Moment of Friction.	f	Moment of Friction / R
A 	$\frac{f F d_2}{2}$	.05	$\frac{.04 P d_2}{p}$
B E 	$\frac{2 f E r}{3}$	.05	$\frac{.2 P r}{p} = \frac{.1 P d_2}{p}$
C E 	$\frac{2 f E (r^3 - r_1^3)}{3 r \sin. y}$	.05	$\frac{.2 P (r^3 - r_1^3)}{p r \sin. y}$
D E 	$\frac{2 f E (r_1^3 - r^3)}{3 (r_1^3 - r^2)}$	.05	$\frac{.2 P (r_1^3 - r^3)}{r_1^2 - r^2}$

or a smaller diameter of worm for a given thread angle, thus keeping within the limits of good practice, and increasing the efficiency of the system for the forward movement.

Having determined the force P_2 tangential to the worm pitch line, resulting from the frictional moment at the journals, the angle of repose for this force acting with the force Q, as shown in Fig. 37, is given by the equation,

$$\tan x = \frac{P_2}{Q}.$$

The thread angle found previous to the consideration of the effect of the journal friction may now be increased by the angle x, making the thread angle $a + x$. This may be accomplished either by increasing the thread angle, increasing the pitch, or decreasing the pitch diameter.

Consider, now, that in the foregoing example, the worm-shaft is of

the form in Case 2, the worm being central between the bearings, and the distance between bearings being 36 inches.

Then, from (5) we have,

$$F = 0.259 \times 5{,}000 = 1{,}295 \text{ pounds,}$$

and from (6)

$$E = 0.966 \times 5{,}000 = 4{,}830 \text{ pounds.}$$

From (10)

$$M = \frac{0.259 \times 5{,}000 \times 36}{4} = 11{,}655 \text{ inch-pounds.}$$

Assuming $s = 10{,}000$ pounds per square inch for the allowable fiber stress in the worm-shaft, we have

$$M = \frac{\pi}{32} d_2^3 s \text{ or } d_2 = 2.28 \text{ inches.}$$

From the table, Case A,

$$P_1 = \frac{0.04 \times 199 \times 2.28}{1} = 18.15 \text{ pounds.}$$

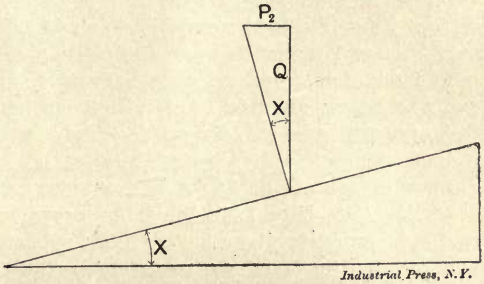


Fig. 37

From the table, Case B,

$$P_1 = \frac{0.1 \times 199 \times 2.28}{1} = 45.37 \text{ pounds.}$$

Then

$$P_2 = 18.15 + 45.37 = 63.52 \text{ pounds.}$$

$$\tan x = \frac{63.52}{5{,}000} = 0.0127$$

$$x = 0 \text{ deg. } 44 \text{ min.}$$

From (1)

$$\tan a = \frac{1}{3.14 \times 8} = 0.04$$

$$a = 2 \text{ deg. } 17 \text{ min.}$$

Then

$$a + x = 3 \text{ deg. } 1 \text{ min.}$$

$$\tan 3 \text{ deg. } 1 \text{ min.} = 0.053$$

$$\frac{p}{\pi d} = 0.053 = h, \text{ and } d = 6 \text{ inches, approx.}$$

If, now, we substitute these new values of h and d in equations (7) and (4), we continue as follows:

From (7)

$$P_1 = 5{,}000\ \frac{0.053 + 0.05}{1 - (0.053 \times 0.05)} = 516 \text{ pounds.}$$

From (4)

$$5{,}000 = \frac{P \times 3.14 \times 6}{1}, \text{ or } P = 265 \text{ pounds.}$$

From (8)

$$\frac{P}{P_1} = \frac{265}{516} = 51 \text{ per cent efficiency for the worm-gear.}$$

The total efficiency of the system, taking account of the journal friction, will be

$$\frac{P}{P_1 + P_2} = \frac{265}{516 + 63.52} = 46 \text{ per cent.}$$

It thus becomes clear that while the efficiency of the worm threads and wheel teeth has been increased above 50 per cent, the efficiency of the whole system, including the journals, is below 50 per cent, and the system retains its self-locking property. It is evident that when running forward, the end thrust E upon the worm-shaft will be upon the opposite end from that when running backward, and on this account a system may be designed to have a high efficiency on the forward movement and still preserve its self-locking property.

If both the journals have roller bearings, and the end taking the thrust on the forward movement has a ball bearing, while the opposite end be made like Case C or D in the table, properly proportioned, the worm may be designed to show a very high efficiency on the forward movement, while the frictional resistance of the step bearing on the opposite end will cause the system to be self-locking by reason of the energy absorbed at the step bearing.

The formulas may be put into more convenient form for this purpose, as follows:

The designer will have, to start with, a knowledge of the force Q required at the worm-wheel, the force P_1 at the pitch line of the worm, developed from the source of power, the pitch required for the worm-wheel, and the efficiency e for which he wishes to design the system. We then have,

$$\frac{P}{P_1} = e, \text{ and } P = P_1 e.$$

Substituting this value for P in equation (4) and solving for d, we have

$$d = \frac{p\,Q}{3.14\,P_1 e}$$

for the worm, neglecting the journals, when the journals and thrust bearings are roller and ball bearings, respectively, and

$$d = \frac{p\,Q}{3.14\,(P_1 - P_2)\,e}$$

when the journals and thrust bearings are considered.

The worm being thus designed for the given efficiency on the forward movement, it remains to determine such proportions of the step bearing for the backward movement as will present enough frictional resistance to render the system self-locking. Let e_1 = the efficiency when the journals and thrust are considered, then

$$\frac{P}{P_1 + P_2} = e_1 \text{ or } P = e_1 (P_1 + P_2)$$

and substituting the value of P found above

$$e P_1 = e_1 P_1 + e_1 P_2$$

and

$$P_2 = \frac{P_1 (e - e_1)}{e_1}.$$

By equating this force P_2 to the proper quantity from Column 4 in the table of journal resistances, the proportions required of the journal or step bearing may be determined.

Theoretical Efficiency of Worm Gearing

The following table gives the theoretical efficiency of worm gearing for a number of different coefficients of friction. Practical experiments carried out by the Oerlikon Company, Oerlikon by Zürich, Switzerland, agree closely with the results from theoretical calculations given in the table. These experiments indicate that the efficiency increases

TABLE GIVING THEORETICAL EFFICIENCY OF WORM GEARING.

Coefficient of Friction.	Angle of Inclination.								
	5 deg.	10 deg.	15 deg	20 deg.	25 deg.	30 deg.	35 deg	40 deg	45 deg
0.01	89.7	94.5	96.1	97.0	97.4	97.7	97.9	98.0	98.0
0.02	81.3	89.5	92.6	94.1	95.0	95.5	95.9	96.0	96.1
0.03	74.3	85.0	89.2	91.4	92.7	93.4	93.9	94.1	94.2
0.04	68.4	80.9	86.1	88.8	90.4	91.4	92.0	92.2	92.3
0.05	63.4	77.2	83 1	86.3	88.2	89.4	90.1	90.4	90.5
0.06	59.0	73.8	80.4	84 0	86.1	87.5	88.2	88.6	88.7
0.07	55.2	70 7	77 8	81.7	84.1	85 6	86.4	86.9	86 9
0.08	51.9	67.8	75.4	79 6	82 2	83.8	84.7	85.2	85.2
0.09	48.9	65.2	73.1	77.6	80.3	82.0	83.0	83.5	83.5
0.10	46.3	62.7	70.9	75.6	78.5	80 3	81.4	81.9	81.8

with the angle of inclination, up to a certain point. They also show that for larger angles of inclination than 25 degrees to 30 degrees the efficiency increases very little, especially if the coefficient of friction is small, and this fact is of importance in practice, because, for reasons of gear ratio and conditions of a constructive nature, an angle greater than 30 degrees cannot be employed. The coefficient of friction increases with the load and diminishes to a certain extent with increase of speed. Besides the friction between the worm and the wheel teeth, there is also the friction of the spindle bearings and the ball bearings for taking the axial thrust. To obtain the best results,

there must be very careful choice of dimensions of teeth, of the stress between them, and the angle of inclination. To show what can be done, the following are the results of a test with an Oerlikon worm-gear for a colliery winding engine: The motor gave 30 brake horse-power to 40 brake horse-power at 780 revolutions. The normal load was 25 brake horse-power, but at starting it could develop 40 brake horse-power. The worm-gear ratio was 13.6 to 1, the helicoidal bronze wheel having 68 teeth on a pitch circle of 7.283 inches, and the worm 5 threads. The power required at no load for the whole mechanism was 520 watts, corresponding to 2.8 per cent of the normal. The efficiency at one-third normal load gave 90 per cent, at full load 94½, and at 50 per cent overload 93 per cent. The efficiency of the *worm and wheel* alone is higher, and knowing the no-load power, is calculated to be 97½ per cent. According to the table given, of theoretical efficiencies, this gives the coefficient of friction as 0.01. To obtain a reduction of 13.6 to 1 with spur gears would have necessitated two pinions and two wheels with their spindles and bearings, and if the bearing friction was taken into consideration, the efficiency of such gearing would certainly not have reached the above-mentioned figure of 94½ per cent at full load. These figures, of course, seem very high for the efficiency of worm gearing. They were published in MACHINERY, December, 1903, having been obtained from a reliable source, and were never challenged.

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